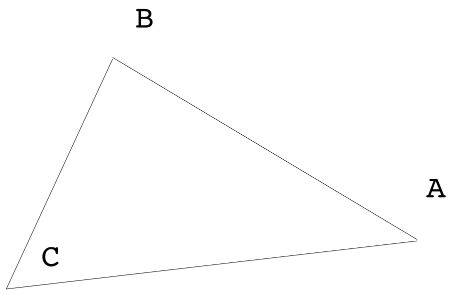
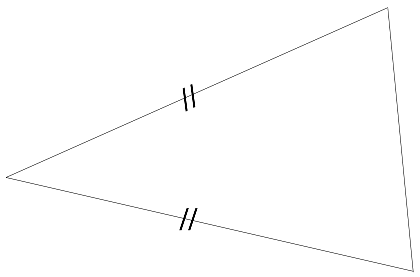
I Triangles



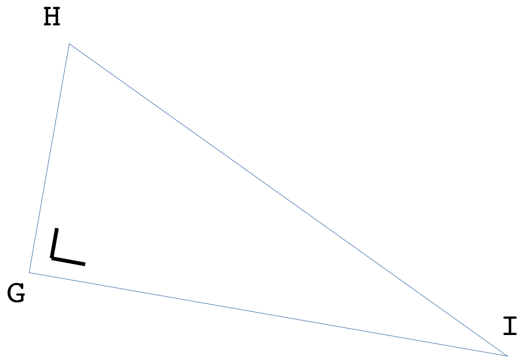
ABC is a triangle, A is a **vertex** /ˈvɜː.teks/ of that triangle, B and C are its two other **vertices** . Since all three interior angles of this triangle are less than 90°, it is an **acute** triangle.

[AB], [BC] and [CA] are the three **sides** of the triangle. Here, the lengths of the three sides are 6 cm, 5 cm and 4 cm. Since all sides are unequal, it is a **scalene triangle** /ˌskeɪ.liːnˈtraɪ.æŋ.gl/.



If two sides of a triangle **are the same length**, the triangle is **isosceles**. /aɪˌsɒs.ᵊl.iːz/ . The angles **opposite** the equal sides are also equal in measure.
In order to draw an __________ triangle, you need a **ruler** and a **pair of compasses**. It is also possible to draw one with the help of a ruler and a **protractor** to ensure that two of the angles are equal in measure.

If the three sides of a triangle are the same length, the triangle is **equilateral**. The three interior angles measure 60° in an ____________ triangle.



If two sides [GH] and [GI] of a triangle are _____________ (or intersect at right angle), the triangle is a **right-angled** triangle **at** vertex G.

The side opposite the right angle is called the **hypotenuse** /haɪˈpɒt.ən.juːz/ .
It is always the longest side.

In order to draw a _____________ triangle, you need a ruler and a **try square** (or a **square**).

A **median** or **midline** of the triangle is a line joining a ______ of a triangle to the ________ of the opposite side. The triangle has three _______, which intersect at a single point. In other words, they are **concurrent**. This point is called the **centroid** of the triangle.

A line drawn from a vertex perpendicular to the opposite side is called a **height** or an **altitude**. The three ________ are also ___________ in a point called the **orthocenter** of the triangle.

The ______________ ___________ of the three sides are concurrent as well. They __________ at the center of the **circumcircle** (the circle **circumscribed** to the triangle). This point is called the **circumcenter**.

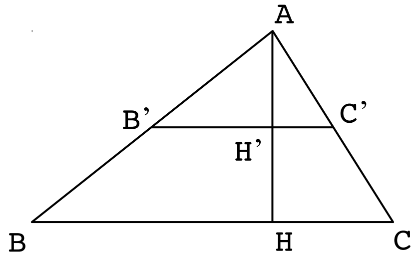
The **bisectors** of the three interior angles intercept at the **incenter**, which is the center of the **incircle** (the circle **inscribed in** the triangle).

Given a triangle, it is possible to calculate its **perimeter** (the sum of the lengths of the three sides) and its **surface area**. The surface area of a triangle is equal to its **base** multiplied by half its **perpendicular height** (*i.e.* the height perpendicular to its base).

Vocabulary: acute triangle – altitude – base – to be the same length – bisector – centroid – circumcircle – circumcenter – circumscribed – (pair of) compasses – concurrent (in ...) – equilateral – height – hypotenuse – incenter – incircle – inscribed (in ...) – isosceles – median – midline – opposite – orthocenter – perimeter – perpendicular height – protractor – right-angled (at ...) – (straight) ruler – scalene triangle – side – (try) square – surface area – vertex *(vertices)*

Exercises

1. The midpoint theorem



In a ________ ABC, B' and C' are the ____________ of the ______ ___________ [AB] and [AC].
The line segment [B'C'] joining the _________ of two ______ [AB] and [AC] is parallel to the third ____ and its length is equal to ____ the ______ of the third side.

2. Heights and feet

H is the ***foot*** *of the ______ through A in the triangle ABC. (AH) and (B'C') intersect at H'.*
Since (BC) and (B'C') are parallel, (AH) is the height ________ A in the triangle AB'C', since "if two lines are ________, any perpendicular to one of them is also _____________ to the other one".
Therefore, H' is the ____ of the height _______ A in the triangle AB'C'. Since [BH] and [B'H'] are ________ and B' is the __________ of [AB], H' is the __________ of [AH].
Also, since the base of the triangle ABC is ______ as long as the ______ of the triangle AB'C', and the height of the triangle ABC is twice as long as the _______ of the triangle AB'C', the _______ area of the triangle AB'C' is a _______ of the _______ ____ of the triangle ABC.
Therefore, the surface area of the ______________ BB'C'C is 3 times the surface area of the ________ AB'C'

3. Pythagoras' theorem

a) *Since H is the _____ of the _______ through A in the triangle ABC, AHB is a right-angled _________ at vertex H. Therefore,* $AB^2 = AH^2 + BH^2$ *, since:*
"In a ______-_______ triangle, the sum of the ________ of the lengths of the ______ containing the right angle is equal to the square of the ______________ ."

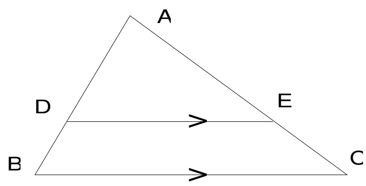
b) *Conversely, if the sides* ***a***, ***b*** *and* ***c*** *of a triangle are such that* $a^2 = b^2 + c^2$ *, then the angle opposite the side of length* ***a*** *is a _______ _______ .*
Prove that a triangle the sides of which measure 5, 12 and 13 centimeters is right-angled.

c) *Using the triangle given in exercise 1, knowing that AH = 15 cm, AC = 17 cm and the surface area of the triangle AH'B' is equal to 37.5 cm², work out the lengths of all the line segments shown in the _________ above.*
Prove *or* ***disprove*** *: ABC is a right-angled triangle at vertex A*

II The **INTERCEPT THEOREM** and its CONVERSE

<u>The intercept theorem</u>:	A straight line drawn parallel to one side of a triangle divides the other two sides in the same ratio.

Complete the sentence :



Let ABC be a triangle. If D belongs to (AB), E ___________ (AC) and (DE) __ ___________ ___ (BC), then according to the intercept theorem:

$$\frac{AD}{AB} = \frac{\quad}{\quad}$$

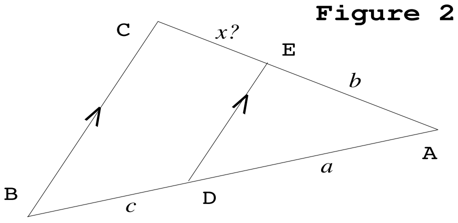
<u>Vocabulary</u>: to disprove – foot – to prove

The converse of the intercept theorem: If a line divides two sides of a triangle in the same ratio, then it is parallel to the third side of the triangle.

Complete the sentence : Let ABC be a triangle. If A, D, B and A, E, C are aligned in the same order and if $\frac{AD}{AB} = \frac{\quad}{\quad}$, then the lines (DE) and (BC) are _______________ .

Exercises: (for the figures, make a **draft** first and then draw the right figure)

Figure 2



1) Using the measurements given in Fig. 2, find the length x of [CE] in terms of a, b and c.

2) ABC is a triangle. A line (PQ) is drawn parallel to (BC), but on the opposite side of A from (BC).

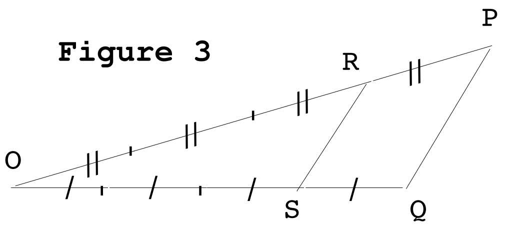
(PQ) cuts (BA) at P and cuts (CA) at Q. AC = 2.5 cm; AQ = 1 cm; AP = 0.5 cm; BC = 5 cm

a) Draw a figure

b) Calculate AB and PQ (explain)

c) Is it possible to draw the right figure? Why?

Figure 3



3) a) Using Fig. 3, give the ratios $\frac{OS}{OQ}$ and $\frac{OR}{OP}$

b) What can you deduce about the lines (RS) and (PQ)? Explain.

4) a) ABC is a triangle, with X a point on [AB] and Y a point on [AC] such that (XY) is parallel to (BC). $AY = \mathbf{3}$ cm ; $YC = \mathbf{4}$ cm and $XB = \mathbf{3}$ cm .

Let H and H' be the feet of the altitudes through A in triangles ABC and AXY. Draw a figure.

b) Calculate AX (explain)

c) Give the ratios $\frac{XY}{BC}$ and $\frac{AH'}{AH}$ (explain).

d) Let $\mathcal{A}$ and $\mathcal{A}'$ be the areas of the triangles AXY and ABC. Calculate $\frac{\mathcal{A}}{\mathcal{A}'}$

5) LMN is a triangle such that MN = 16 cm. Q is a point on [LN] such that LQ = 2 cm and QN = 6 cm. The parallel to (MN) through Q intersects (LM) at P such that LP = 3 cm.

5.1 Draw a figure.

5.2 Calculate MP (explain).

5.3 R is a point on [MN] such that MR = 12 cm. Prove that (PR) // (LN).

5.4 Calculate PR (two different ways exist, at least).