

## I The circle

A **circle** is the locus (or the set) of all the points that are at the same distance, called the **radius** of the \_\_\_\_\_, from a fixed point, called the **centre** (US: *center*) of the \_\_\_\_\_.

[OA] is a radius */ˈreɪ.di.əs/* of the circle, since its end-points are the \_\_\_\_\_ of the circle and a \_\_\_\_\_ that belongs to the circle.

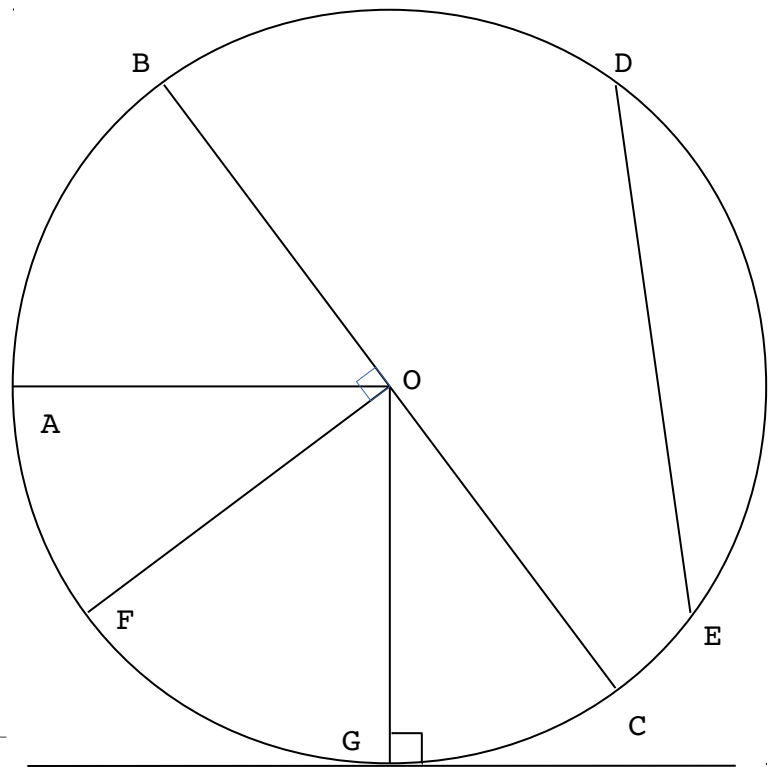
[OB] and [OC] are two **radii** */ˈreɪ.di.ɑɪ/*. Since the \_\_\_\_\_ of the circle belongs to the line segment [BC], [BC] is a **diameter** of the circle.

[DE] is a **chord**. */kɔ:d/*

A \_\_\_\_\_ is a particular \_\_\_\_\_ passing through the centre of the circle.

The line segment [OA] measures 5 cm. Therefore, the \_\_\_\_\_ of the circle is equal to 5 cm.

The \_\_\_\_\_ to (BC) through O intersects the circle at F and F' (not **plotted** on the drawing), therefore, [OF] is also a \_\_\_\_\_ and [FF'] is a \_\_\_\_\_.



Let G be a point of the circle. The \_\_\_\_\_ to the radius [OG] \_\_\_\_\_ G is the **tangent** */ˈtæŋ.dʒənt/* to the circle at G. A \_\_\_\_\_ to a circle is a line that intersects the \_\_\_\_\_ at exactly one point, that is called the point of **tangency**, or the point of **contact**.

### Exercise 1:

*Prove that the perpendicular bisector of any chord passes through the centre of the circle.*

An **arc** of a circle is a part of the circle. If the circle is divided into two unequal parts, the longer of the two arcs is called the **major** (or **long**) arc, and the shortest one is called the **minor** (or **short**) arc. The two form a pair of **conjugate** arcs.

Either of the two parts of the circle cut by the diameter [BC] is a **semicircle**, so  $\widehat{BC}$  is a \_\_\_\_\_.  $\widehat{BF}$ , \_\_\_\_\_ and \_\_\_\_\_ are **quarter circles**, and  $\widehat{BA}$ ,  $\widehat{CE}$  and  $\widehat{DG}$  are arcs of the circle. */ˈsem.ɪ.sɜː.kl/*

The surface **bounded** by the circle, or enclosed by the circle, is called a **disc** (US : *disk*).

The surface bounded by the radii [OB] and [OG] and the arc  $\widehat{BG}$  is a **circular sector**.

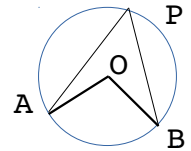
The surface \_\_\_\_\_ the chord [DE] and the arc  $\widehat{DE}$  is a **circular segment**. Actually, the chord [DE] divides the disc into 2 \_\_\_\_\_: the major segment is the region between the chord and the major (or **greater**) arc, and the minor segment between the chord and the minor (or **lesser**) arc.

Vocabulary: arc – to bound – centre – chord – circle – circular sector – circular segment – conjugate – contact – diameter – disc – greater / lesser – long / short – major / minor – to plot – quarter circle – radius (*radii*) – semicircle – tangency – tangent

## Angles and triangles

$\angle APB$  is the angle **subtended by** arc  $\widehat{AB}$  **at** point P on the circumference of the circle.

$\angle AOB$  is the angle \_\_\_\_\_ arc  $\widehat{AB}$  \_\_\_\_\_ point O, centre of the circle.



**Theorem :** The angle that an arc subtends at the centre of a circle is twice the angle that this arc subtends at points on the remainder of the circumference.

**Thales' theorem :** An angle inscribed in a semicircle is a right angle.

**NOTICE** that this theorem is known by the French as : \_\_\_\_\_

**Conversely :** The **circumcentre** of a right angled triangle is the midpoint of the hypotenuse. Therefore, the hypotenuse of a right angled triangle is a diameter of its **circumcircle**.

## Perimeter, length and surface area

Let  $r$  be the radius of a circle and  $d$  its diameter. The perimeter is equal to  $2\pi r$  (also equal to  $\pi d$ ).

The surface area inside the circle is equal to  $\pi r^2$ , which is also equal to  $\frac{\pi d^2}{4}$

$\pi$  is a constant.  $\pi \approx 3.1416$  to 4 decimal places. Its exact value can be expressed neither as a fraction nor as a surd involving square roots of rational numbers.

There are two famous approximations with fractions:

$\frac{22}{7}$  gives two exact decimals, whereas  $\frac{355}{113}$  gives six correct ones.

In order to measure the length of an arc, we need to know the measure of the angle at the centre subtended by the arc. The measure of the angle and the length of the arc are **proportional**.

The area of the \_\_\_\_\_ sector and the measure of the angle are \_\_\_\_\_ as well.

If  $\alpha$  is the measure of the angle in degrees and  $r$  is the radius of the circle, the length  $L$  of the arc and the area  $A$  of the circular sector are given by the following formulas (or formulae) :

$$L = 2\pi r \frac{\alpha}{360} \quad \text{and} \quad A = \pi r^2 \frac{\alpha}{360}$$

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### Exercise 2:

Calculate the perimeter and the surface area of the disc page 1. With your protractor, measure the angles  $\angle FOG$ ,  $\angle GOC$ ,  $\angle COE$  and  $\angle EOD$ . Then calculate the lengths of the 4 minor arcs and the surface areas of the corresponding circular sectors. Round off the results in cm or  $\text{cm}^2$  to 2 d. p.

Why do you think it is useless to give the results with more than 2 digits after the decimal point?

### Exercise 3:

Let  $C$ ,  $S$  and  $T$  be a circle, a square and an equilateral triangle of same perimeter  $P$ . Calculate the side of the square and the triangle, and the radius of the circle. Then compare their surface areas.

Let  $C$ ,  $S$  and  $T$  be a circle, a square and an equilateral triangle of same surface area  $A$ . Compare their perimeters. (Extra question: you may add a regular hexagon in the two previous questions)

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**Vocabulary :** circumcenter – circumcircle – proportional – to subtend ... at

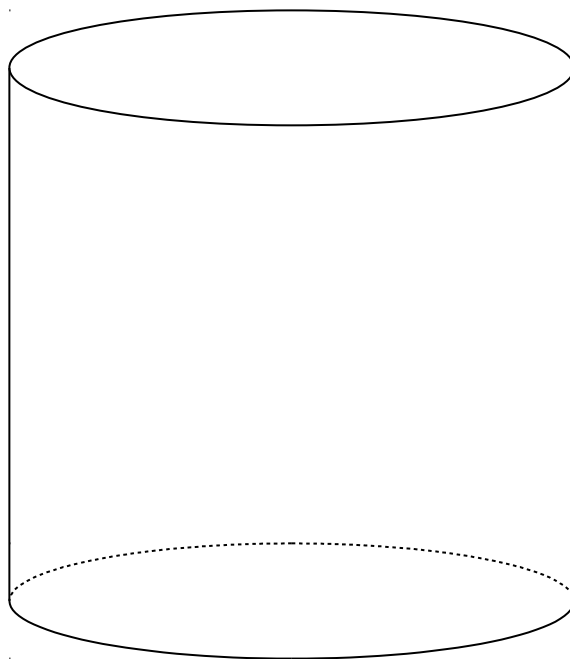
## II Cylinders, cones and spheres.

The solid to the right is an example of a **cylinder** . Since its **bases** are discs, it is a circular cylinder. The **lateral surface** is formed by **generators** joining corresponding points on the perimeters of the bases.

Since the generators are perpendicular to the bases, it is a **right** cylinder. Otherwise it would be an **oblique** cylinder /'sɪl.ɪn.də'/.

The perpendicular distance between the bases is called the **altitude** (or **height**) of the cylinder.

The volume of the cylinder is equal to  $Ah$ , where  $A$  is the surface area of the bases and  $h$  the altitude of the cylinder. Show  $A$  and  $h$  on the drawing.

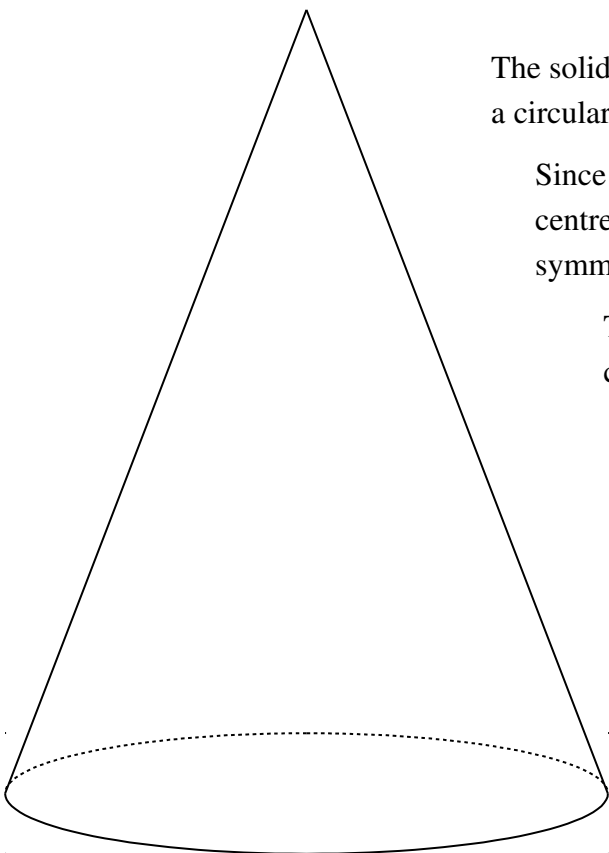


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### Exercise 4:

Calculate the volume of a food can shaped as a cylinder of diameter 11.3 cm and of height 24.0 cm. Round off the result to the number of decimal places you like. Explain your choice.

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The solid to the left is an example of a **cone**. Since its base is a disc, it is a circular cone. The pointed tip is called the **vertex** of the cone.

Since the base has a **centre of symmetry** /'sɪm.ə.tri/ , which is the centre of the circle, the line joining the vertex to that centre of symmetry is the **axis** (*plural* : *axes*) of the cone.

The perpendicular distance between the base and the vertex is called the altitude (or height) of the cone.

Here, the axis is perpendicular to the base. Therefore, it is a right cone. Otherwise it would be an oblique cone.

In a right circular cone, the distance between the vertex and any point of the edge of the base is called the **slant height**. /'slɑːnt haɪt/

The volume of the cone is equal to  $\frac{1}{3} Ah$  , where  $A$  is the surface area of the base and  $h$  the altitude of the cone.

Show  $A$  and  $h$  on the drawing.

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Vocabulary : altitude – axis (*axes*) – base – centre of symmetry – cone – cylinder – generator – height – lateral surface – oblique / right – slant height – vertex

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Exercise 5:

Consider a circular right cone of altitude  $h$ . The radius of its base is  $r$ .

Prove that its slant height (noted  $s$ ) is equal to  $\sqrt{r^2 + h^2}$

Exercise 6 - NET OF A CONE

Consider a circular right cone of altitude  $h$ . The radius of its base is  $r$ .

Let  $\alpha$  be the measure of the angle at the centre corresponding to the circular sector used for the lateral surface of the cone. What is the radius of this circular sector? Express in two different ways the length of the arc which subtends the angle at the centre of measure  $\alpha$ .

Deduce that 
$$\alpha = 360 \frac{r}{\sqrt{r^2 + h^2}}$$

Draw a net of the cone when  $h=8$  cm and  $r=4$  cm.

Prove that the area of the lateral surface is equal to  $\pi r s$ , where  $s$  is the slant height.

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The drawing to the right is a **sphere** /sfiə/ of radius  $r$ .

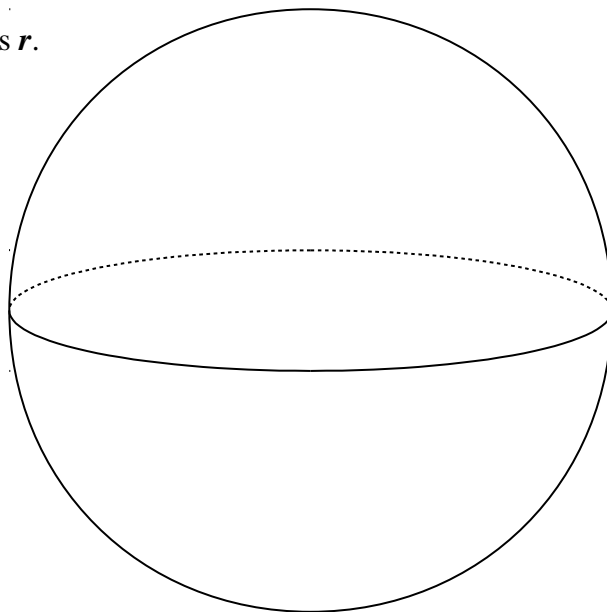
Its (enclosed) volume is equal to  $\frac{4}{3}\pi r^3$  ;

Its surface area is equal to  $4\pi r^2$  .

A **great circle** is a circle, whose points are on the sphere, and that has its centre at the centre of the sphere. Therefore, its radius is equal to the radius of the sphere.

Any other circle on the sphere is a **small circle** of radius less than the radius of the sphere.

Circles on the sphere are obtained as the intersection of a plane and the sphere.



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Exercise 7:

Consider a sphere of radius  $r = 10$  m. Let us paint it with a uniform layer of paint 0.1 mm thick.

What becomes of the radius of the sphere?

What is its new volume (expand the brackets)?

Is there any other way we could have calculated an approximation of the new volume?

Exercise 8:

Are there axes of symmetry in the cylinder? If any, describe them.

Answer the same question with a sphere.

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