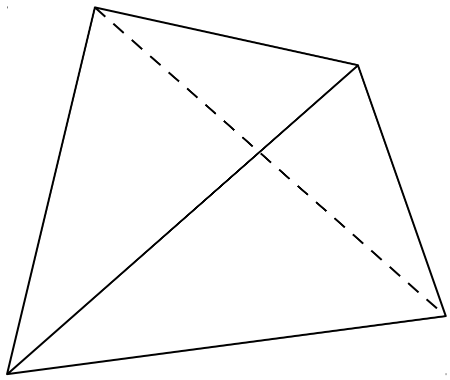
I. Polyhedra

Definition : A **polyhedron** is a solid with a surface composed of polygons, which are called **faces**.
The sides of the polygons are the **edges** of the polyhedron,
whereas a point where three or more faces meet is called a **vertex** /ˈvɜː.teks/

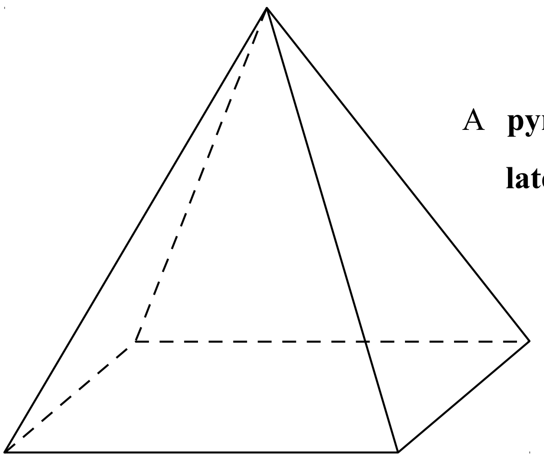
Four is the minimum number of faces in any polyhedron. /ˌpɒl.iˈhiː.drən/

Definition : **congruent** /ˈkɒŋ.gru.ənt/ (adjective, specialized) ,
describes a shape in mathematics that has the same shape and size as another : congruent triangles.

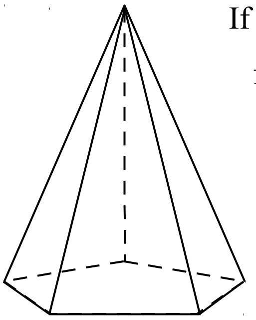
Definition : A **regular** polyhedron has regular congruent faces, that are all at the same angle.



A **tetrahedron** /ˌtet.rəˈhiː.drən/ has four triangular _________ , six _________ and four **vertices** /ˈvɜː.təˌsɪz/ where three faces meet. If the _____________ is regular, its faces are equilateral triangles.



A **pyramid** /ˈpɪr.ə.mɪd/ has a polygonal **base** and triangular **lateral faces** with a common vertex. The line segments from the common vertex to the vertices of the base are the **lateral edges**. Pyramids are named according to the nature of their base: a tetrahedron is a triangular ____________ , the Egyptian pyramids are ________ pyramids.

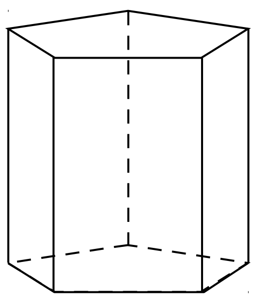


If the base has a centre, a line from the centre to the common vertex of the lateral faces is the **axis** of the pyramid. If the axis is perpendicular to the base, the pyramid is a **right** pyramid, otherwise it is an **oblique** /əʊˈbliːk/ pyramid.
If the base of a right pyramid is a regular polygon, the pyramid is **regular**.

Vocabulary : axis (*axes*) – base – congruent – edge – face – lateral (edge, face) – polyhedron (*polyhedra*) – (oblique, regular, right) pyramid – regular (polyhedron) – tetrahedron – vertex (*vertices*)

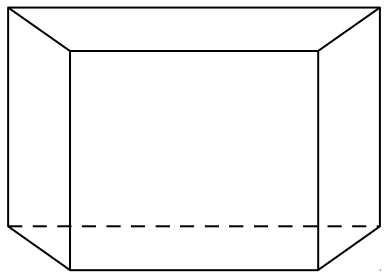
Exercise 1 : Khufu's pyramid is a _________ pyramid, since its ______ is a square of side 232 m, further it is a right pyramid, since its axis is _____________ to the base.

*Knowing that the volume of a pyramid is equal to a third of the area of its base multiplied by the **altitude**, which is the perpendicular distance from the base to the vertex, calculate the volume of Khufu's pyramid, the height of which measures approximately 147 meters.*

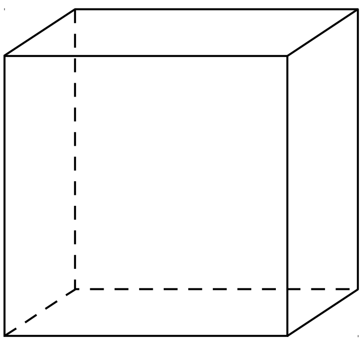


A **prism** /ˈprɪz.ᵊm/ has two __________ polygonal bases with parallel corresponding sides. The lateral faces are parallelograms obtained by joining the corresponding vertices of the bases.

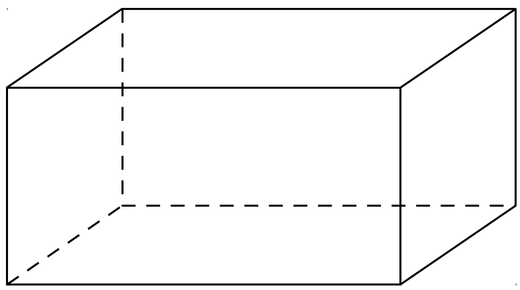
Prisms are also named according to the shape of their bases.



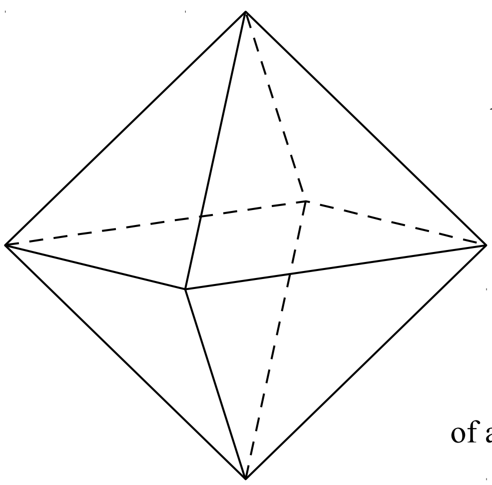
In a **right** prism, lateral edges are at right angles to the bases and lateral faces are rectangles. In an **oblique** prism, this is not the case.



A **cube** is also a regular polyhedron, it has 6 square faces, 12 edges, 8 vertices, and 4 **diagonals** /daɪˈæg.ᵊn.ᵊl/ , since diagonals join two vertices that are not in the same face.



A **cuboid** /ˈkjuː.bɔɪd/ or a **rectangular parallelepiped** has six rectangular faces. It is a right parallelepiped with rectangular bases. Its volume is equal to the product of its length by its breadth (or width) by its height.



A regular **octahedron** has 8 faces which are equilateral triangles, 6 vertices, 12 edges and 3 diagonals.

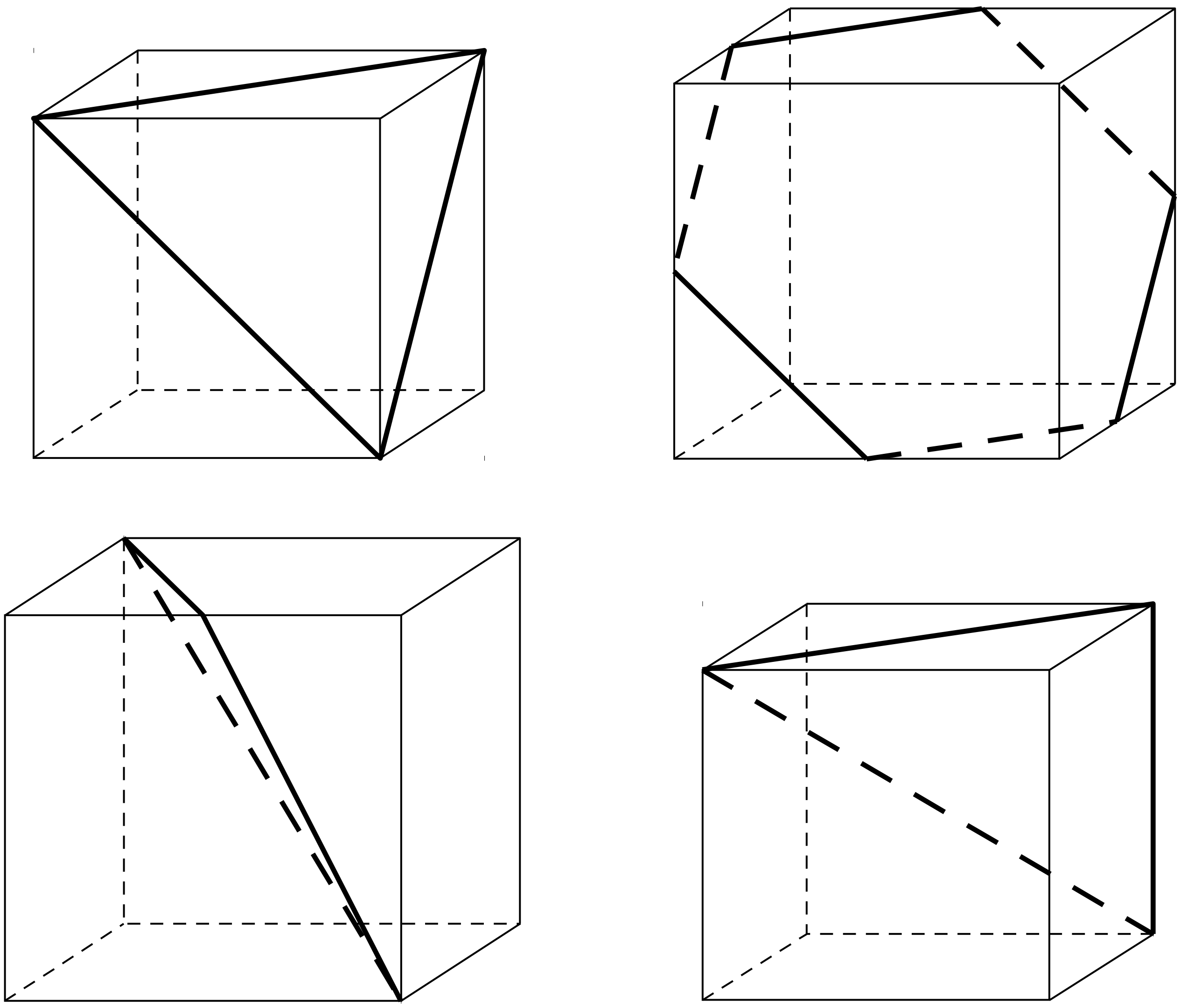
It is the third regular polyhedron.

In order to draw a regular octahedron, you can use the centers of the faces of a cube for the vertices.

In order to draw a regular tetrahedron, you can use four vertices of a cube, chosen in such a way that none are in the same edge.

Vocabulary : altitude – cube – cuboid – diagonal – octahedron – (rectangular) parallelepiped – (oblique, right) prism

<u>Exercise 2</u>:
*The outlined polygons are included in plane **sections** of a cube. What are they? Explain why.*
Which ones are exactly the intersection of a plane and the cube ?



II. Regular Polyhedra

Reminder: a regular polyhedron has identical regular polygons forming its faces and identical vertices. If one of those two conditions lacks, the polyhedron is not a regular polyhedron.

There are only five regular polyhedra, also known as **Platonic solids** :

the tetrahedron, the cube, the octahedron, the **dodecahedron** and the **icosahedron**.

The cube, the tetrahedron and the octahedron were already shown in the previous section.

The fourth regular polyhedron is called the dodecahedron. /ˌdəʊ.dek.əˈhiː.drᵊn/

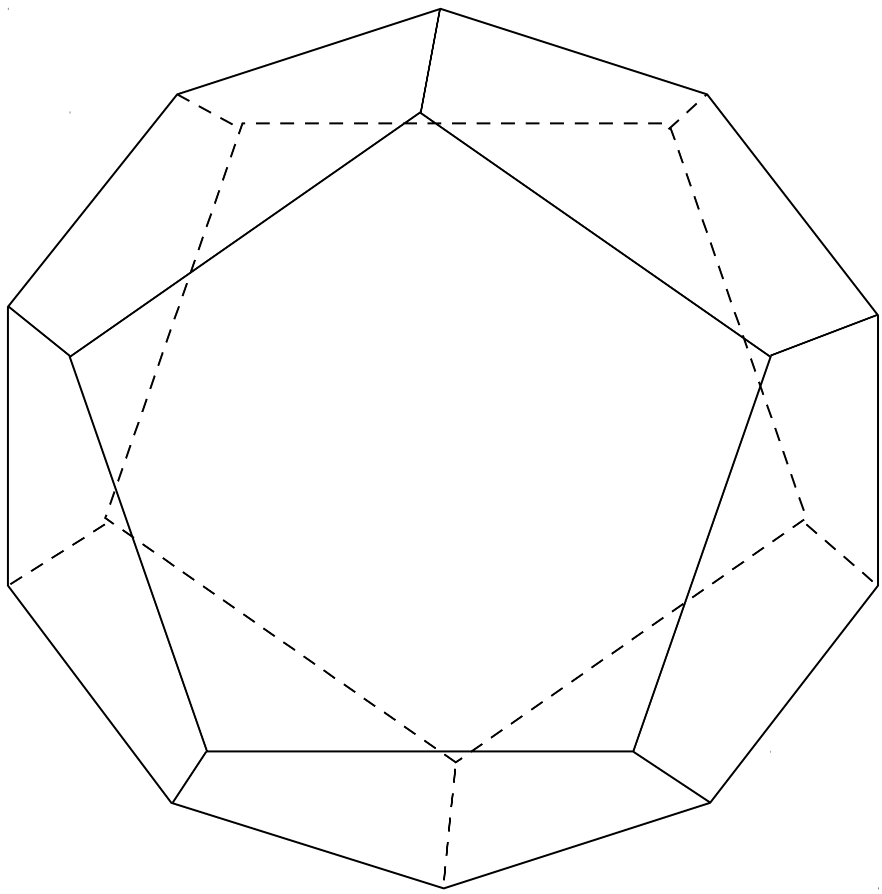
It has twelve **pentagonal** faces, thirty edges and twenty vertices, each vertex being common to three faces.

<u>Vocabulary</u> : dodecahedron – icosahedron – pentagonal (face) – Platonic solid – section

Exercise 3:

Using the view of the regular dodecahedron, determine how many diagonals there are in a regular dodecahedron. Bear in mind that a diagonal in a polyhedron is a line segment that joins a vertex of a polyhedron to another vertex that is not in the same face. A line segment that joins two vertices in the same face is either an edge, or a diagonal of that face.

(hint: count how many diagonals there are through one vertex first).



Since polyhedra has plane faces, and straight edges, it is possible to cut the surface of a polyhedron following some of its edges, and to unfold by **rotating about** the remaining uncut edges : the resulting plane figure is a **net** for the polyhedron.

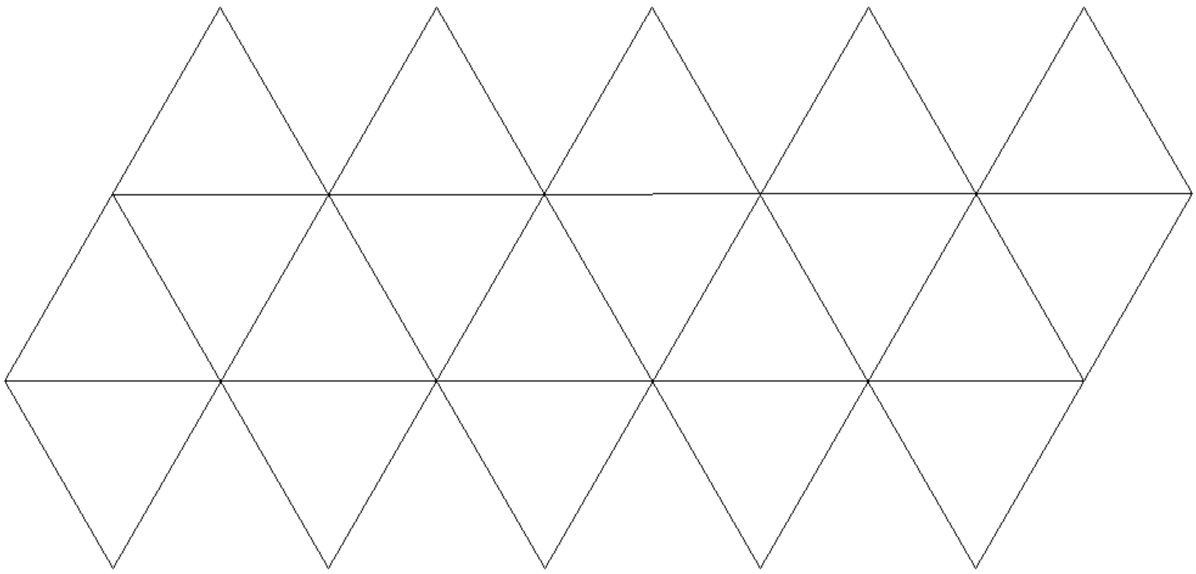
Exercise 4:

*Using a pair of compasses, **sketch** a net for a dodecahedron.*

Vocabulary : net – to rotate about ... – to sketch

A regular icosahedron has twenty equilateral triangles for faces, thirty edges and twelve vertices.

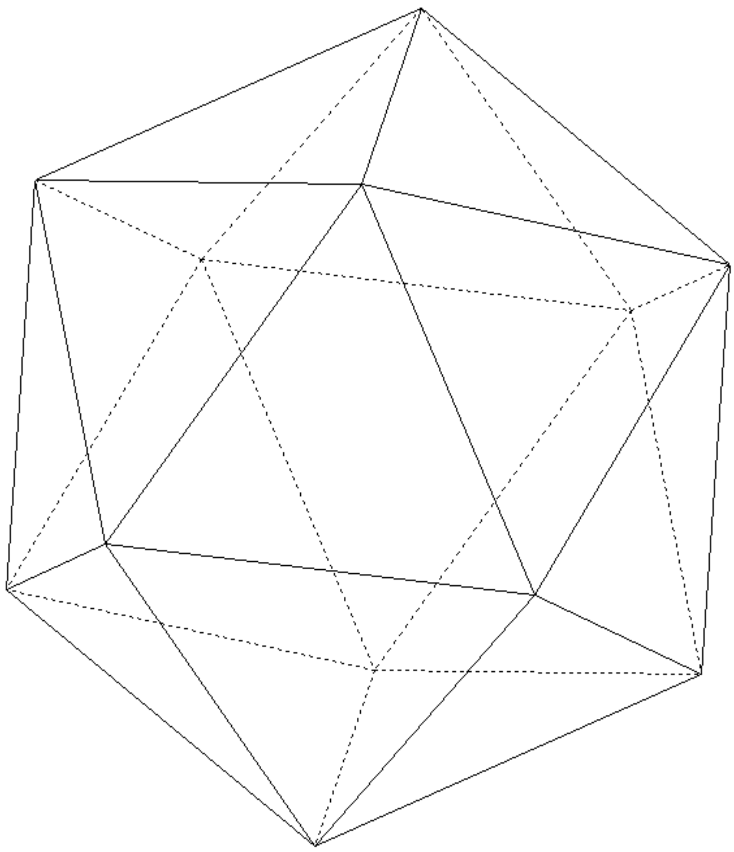
This is a net for an icosahedron. /aɪˌkəʊ.səˈhiː.drᵊn/



It could be used to draw a fairly accurate map of the Earth.

<u>Exercise 5</u> :

Using the view below, determine how many diagonals there are in a regular icosahedron.



<u>Exercise 6</u> :

The town council has a new swimming pool built. The pool is a rectangular parallelepiped of length 50 *meters, of breadth* 16 *meters and of height* 2.4 *meters. The water level in the pool is* 2 *meters.*

- *Calculate the amount of water required to fill in the pool. A water hose can pour 5 liters per minute; a fire hose can pour 200 liters per minute. How long would it take to fill in the pool with a water hose or with a fire hose?*
- *The bottom, the lateral sides of the pool and a 4 meter wide gangway around the pool have to be tiled with ceramic tiles 10 cm long by 5 cm wide. How many tiles are required in total?*

<u>Exercise 7</u>: Calculation of a regular tetrahedron's volume.

ABCD *is a regular tetrahedron of base* ABC. *All the edges of the tetrahedron measure* $\boldsymbol{a}$ = 10 *cm.*

a) *Prove that the height of an equilateral triangle of side a measures* $a\dfrac{\sqrt{3}}{2}$

b) *Prove that the surface area of any face of the tetrahedron is equal to* $a^2\dfrac{\sqrt{3}}{4}$

c) *Let* H *be the foot of the altitude through* D *and let* I *be the midpoint of* [BC]. *We will admit that* A, D, H *and* I *are coplanar, i.e. lie in the same plane.*

- *Looking at a top view of the tetrahedron (1st drawing underneath),* H *and* D *are exactly superimposed. Deduce the position of* H *in the triangle* ABC. *Can you prove what you think?*
- *What is the nature of the triangle* ADI *(2nd drawing underneath)?*
- *If the edge of the tetrahedron measures* $\boldsymbol{a}$, *prove that* $AH = a\dfrac{\sqrt{3}}{3}$ *and that* $HD = a\dfrac{\sqrt{2}}{\sqrt{3}}$
- *Hence deduce that the volume* $\boldsymbol{V}$ *of the regular tetrahedron is :* $V = \dfrac{\sqrt{2}}{12}a^3$
- *If the tetrahedron vertices are 4 of the vertices of a cube of side* $\boldsymbol{b}$, *prove that:* $V = \dfrac{1}{3}b^3$