1. Monotonic functions

Graphs are read from left to right.

Definition : A **monotonic** (or **monotone**) function changes always in the same direction.

A description : the function f is **decreasing** where its graph is falling as we go from left to right.

... or a definition : A **monotonic decreasing** function is a function *f* with domain and **codomain** (or range) that are sets of real numbers, such that the output value decreases or stays the same as the input value increases.

Formally : Let [***a*** , ***b***] be a subset of the domain, part of $\mathbb{R}$.
If for every x_1 and x_2 such that $a \leqslant x_1 < x_2 \leqslant b$, we have $f(x_1) \geqslant f(x_2)$,
f is said to be monotonic decreasing on [***a*** , ***b***].

If we have $f(x_1) > f(x_2)$, f is said to be **strictly** monotonic decreasing on [***a*** , ***b***].

The interval [***a***,***b***] is a set of values of ***x*** for which ***f(x)*** is decreasing.

The function f is increasing where its graph is rising as we go from left to right.

... or a definition : A **monotonic increasing** function is

The interval [***a***,***b***] is a set of values of ***x*** for which ***f(x)*** is increasing.

examples

Vocabulary codomain – decreasing – increasing – monotone – monotonic – strictly

2. Local maximum and minimum

Often we are asked to find the local maximum and minimum points (of a graph), or the local maximum and minimum values (reached by a function).

<u>Definition</u> : The **minimum** (*plural* minima) value of a function is the **least** value that it **attains**.

A **point** (*or* a number) ***x*** is a **local minimum** of a function ***f*** if $f(x) \leqslant f(y)$ for all points (*or* for all numbers) ***y*** in a **neighbourhood** of ***x*** (*or* ... in an interval containing ***x***, and numbers greater than ***x***, and numbers less than ***x***).

x is a **strict local minimum** if $f(x) < f(y)$ for all ***y*** in a neighbourhood.

<u>Definition</u> : The **maximum** (*plural* maxima) value of a function is

x is a **strict local maximum** if

<u>examples</u>

3. Graphing the absolute value function

The **absolute value** (or **modulus**) of a number ***x***, written $|x|$, is the positive value of ***x*** disregarding its sign.
Then $|x|$ is the square root of ___ ; it is equal to ***x*** if ***x*** is __________ and to ____ if ***x*** is ___________ .
If $f(x)=|x|$, its domain is _____ , its range is _____ and ***f*** is an _________ function,
then its graph $\mathcal{C}$ is _________________________________ .

figure 1 – graph of $|\,\mathbf{x}\,|$

<u>*Exercise 1*</u> – $f(x)=|2-x|$ *Using a sign-table, express* ***f(x)*** *without modulus* ***within*** *2 intervals and draw its graph* C *in an orthonormal system of axes, taking 1 cm to 1 unit. What do you notice about* C*?*

<u>*Exercise 2*</u> – $g(x)=4-|x|$ *What do you notice about* ***g*** *and what can you deduce about its graph* C'*? Draw* C' *in the same system as* C*.*

<u>*Exercise 3*</u> *– Shade the area in which a point* P *can lie if its coordinates satisfy the system of simultaneous inequalities :*

$$\begin{cases} y \geqslant |2-x| \\ y \leqslant 4-|x| \end{cases}$$

<u>Vocabulary</u> absolute value – greatest – least – local – maximum (maxima) – minimum (minima) – modulus – neighbourhood – strict – to attain