QUADRATIC EQUATIONS

COMPLETING THE SQUARE

I. Definition

Any equation of the form $ax^2+bx+c = 0$ with $(a \neq 0)$ is called a **quadratic equation**.
The values of x that satisfy the equation are called the **roots** of the equation.
A quadratic equation has at most two roots, called generally x' and x'' when they exist.
If $x'=x''$, the equation is said to have a **repeated root**.
If the equation has no root, the set of solutions is the **empty set** ($\varnothing$)

Exercise 1

Solve the quadratic equations and write the conclusions :

a) $x^2-5x+\frac{25}{4} = 0$ *(write the LHS as a perfect square)*

b) $x^2+9 = 0$

c) $9x^2-36 = 0$ *(write the LHS as a difference between two squares)*

II. Solution of quadratic equations by completing the square

Exercise 2 – Consider the quadratic equation : $x^2-6x+5 = 0$ (I)

1. Find the values of a and b completing the square $x^2-6x+b = (x-a)^2$,
   then, substituting this expression of x^2-6x , write equation (I) in the form $(x-a)^2-c = 0$.
2. Factorising the LHS as a difference between two squares, solve the equation (I), and make a sentence.
3. Using a **sign-table**, state the range of values of x for which the inequality $x^2-6x+5 \geqslant 0$ is true.
   (the symbol for **infinity** is ∞ , and the symbol for **union** is $\cup$)

Exercise 3 – By completing the square, solve the following equation : $x^2+4x-5 = 0$
and find the values of x that satisfy the inequality : $x^2+4x-5 < 0$

Exercise 4 – Peter had x marbles. The number of marbles Fred had was six fewer than the square of the number Peter had. Together they had 24 marbles.

Form a quadratic equation in x and solve it using the previous method. How many marbles did Fred have ?
Check your answer.

Exercise 5 – Assume the width x and the length y of a rectangle are integers.
This rectangle is 6 cm longer than it is wide, and its area is less than the area of a square of side 4 cm.

1. Write an inequality of the form $f(x)<0$ where $f(x)$ is a quadratic expression.
2. Using the previous method, factorise $f(x)$.
3. Using a sign-table, find the range of values of x that satisfy the inequality.
4. Find the dimensions of the rectangle.

Check your answers.

Vocabulary empty set – infinity – quadratic (equation) – repeated root – root – sign-table – union