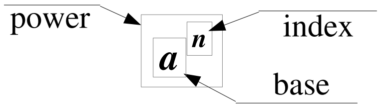
I. Powers

Definition :

When a number is to be multiplied by itself repeatedly, the result is a **power** of this number.



r^2 reads "*r* **squared**" or "*r* (**raised) to the power of** 2" or "*r* to the second power" /paʊəʳ/.

a^3 reads "*a* **cubed**" or "*a* (raised) to the power of 3" or "*a* to the third _________"

2^n reads "two ___ the _________ ___ *n*" or "two ___ the ____ power".

The number c^n is a power in which *c* is the **base** and *n* is the **exponent** or the **index** (plural: indices).

If *b* is a non zero real number and *n* an integer, $\frac{1}{b^n}$ is the **reciprocal** (or the **multiplicative inverse**) of b^n. It should not be mistaken with the **opposite** of b^n, which is $-b^n$. Opposite numbers usually have **unlike signs**.

$\frac{1}{2}x^2$ reads "half *x* squared". $4x^2y^3$ reads "four *x* ________ *y* ________".

$2^3=8$ means that 8 is the third power of 2; 2 is the base, 3 is the exponent. /ɪkˈspəʊ.nənt/

If *n* is an _____ integer, *a* and a^n always have like signs. If *n* is ______, a^n is a non negative number.

II. Algebraic expressions

When we perform mathematical operations with expressions that include letters, we call it algebra.

Variable: Letters that are used to stand for different numbers are called **variables** or **unknowns**.

Constant: Anything that has a fixed value is a **constant** /ˈkɒnt.stᵊnt/. Examples: 2 or π are _________.

Term: A **term** is the product of some constants and ____________.
Example: if we calculate the surface area *S* of a cylinder of radius *r* and height *h*, $S=2\pi r^2+2\pi rh$. The surface area is the _____ of two ______, $2\pi r^2$ and $2\pi rh$.

Coefficient: The constant part of a term is called the **coefficient** /ˌkəʊ.ɪˈfɪʃ.ᵊnt/ of the term.
In the example above, the ___________ is 2π in both terms.

Expression: One single term or a collection of terms separated by **+** or **–** signs. Still in the same example, we write that the variable *S* is equal to the **expression** $2\pi r^2+2\pi rh$.

Like terms: Terms that use the same variable to the same power, or the same product of such powered variables, are **like terms**. The only difference between ____ _____ is the terms coefficients.
Example: $2x$; $-3x$ and πx are _____ terms, whereas $3x$ and $3y$ are unlike ______.

Usually, in every term, we write first the constant expressed in figures, then the other constants, then the variables. In any given expression, it is best to write the variables always in the same order.

Exercise 1: Simplify the different expressions as in the example below. Describe what you do.

Example:

$3xy+5yz-4zx+2yx = 3xy+5yz-4xz+2xy$ (*rewrite in the same order*)

$= 3xy+2xy+5yz-4xz$ (*group like terms together*)

$= 5xy+5yz-4xz$ (*simplify like terms*)

a) $2x^2+3x+4+3x-4x^2-5$ b) $2x^3-8x^2+6x-x^2+5x-3$ c) $3x^3-6x+4x^2-2+x^2+5x$

Vocabulary: base – coefficient – constant – to cube – exponent – expression – index (*indices*) – multiplicative inverse – like (signs / terms) – opposite – ((to raise) to the) power (of) – reciprocal – to square – term – unknown – unlike (signs / terms) – variable

III. Expanding and factorising

If an algebraic expression contains brackets, it is possible to get rid of them by **expanding** the **brackets**. In order to do so, do the following:

- Multiply each term inside the brackets by the number outside the ___________ ,
- Any term not in the brackets is left unchanged,
- Simplify by rewriting variables in the same order in all terms, collecting and adding _____ _______ .

On the other hand, it is often convenient to **factorise** (US : factorize) an expression. Factorising is the reverse procedure to removing brackets. It is done by **taking out** the common factors.

Example:	Expanding:	$(3x-5)(2x+1) = 6x^2-7x-5$		*Remove brackets*
	Factorising:	$6x^2-7x-5 = (3x-5)(2x+1)$		*Put in brackets*

If possible, factorise also the constants. Example: $3p^2-12p+12 = 3(p^2-4p+4) = 3(p-2)^2$

There are three **special factors** or **special expansions** that you should know:

- $(a+b)(a-b) = a^2-b^2$ This special expansion is called the **difference of two squares**.
- $(a+b)^2 = a^2+2ab+b^2$
- $(a-b)^2 = a^2-2ab+b^2$

<u>*Exercise 2:*</u> *Prove the following four equalities by expanding the brackets.*

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$	$(a-b)^3 = a^3-3a^2b+3ab^2-b^3$
$a^3+b^3 = (a+b)(a^2-ab+b^2)$	$a^3-b^3 = (a-b)(a^2+ab+b^2)$

<u>*Exercise 3:*</u> *Remove the brackets and then simplify the following expressions.*

a) $2(1-3x-x^2)-(2-6x-3x^2)$ *b)* $5(2x^2-3x+2)-3(3x^2-6x+2)$

c) $4x-[4x-2x(2x-2)]$ *d)* $2x(3x^2-6x+2)-3(2x^2-8x-4)$

e) $(x+2)(3x^2+2x+4)$ *f)* $(2x-5)(2x^2+2x-3)$

<u>*Exercise 4:*</u> *Factorise the following expressions as much as possible. If necessary, expand first.*

a) $6xy+3y^2$ *b)* $5x^2-20y^2$

c) $(x+2y)^2+x^2-4y^2$ *d)* $ac-2bd-ad+2bc$

<u>*Exercise 5:*</u> *Factorise each of the following using the difference of two squares.*

a) x^2-9 *b)* $4x^2-9$

c) $36-25x^2$ *d)* $81x^2-\frac{1}{16}$

<u>Vocabulary</u>: brackets – difference of two squares – to expand – to factorise – special factor – special expansion – to take out

IV. Evaluating expressions

When we replace letters with numbers when **evaluating** expressions, we call it **substitution**. It is often a good idea when ____________ a number **for** a variable in an algebraic expression to rewrite the expression, putting a bracket around the number that replaces the letter, and then calculate.

Example : Evaluate the expression $A=x^2+y^2$ if $x=-2$ and $y=1$.

Rewriting A : $A=(-2)^2+(1)^2=4+1=5$

V. Multiplication

The index tells you how many times a number is multiplied by itself.

$a\in\mathbb{R}$ $n, p \in\mathbb{Z}$ $\boxed{a^n a^p = a^{n+p}}$: to multiply powers of the same number, add the indices

<u>example</u> : Simplify $2c^2d\times 3c^3d^2$

working $2\times 3=6$; $c^2c^3=c^{3+2}=c^5$; $dd^2=d^{1+2}=d^3$

therefore $2c^2d\times 3c^3d^2 = 6c^5d^3$

<u>Exercise 6</u> : *Simplify, then read* :

① $3a^2\times 4a^3$ ② $2x^2y\times 3xy$ ③ $2a^2b^2\times 5b^2$

④ $6x^3\times 4x^3y^3$ ⑤ $-2x^2\times 3x^4$ ⑥ $-5xy^2(-5x^2y)$

VI. Division

$a\in\mathbb{R}$ $n, p \in\mathbb{Z}$ $\boxed{\dfrac{a^n}{a^p} = a^{n-p}}$: to divide powers of the same number, subtract the indices

If a positive index is required : if $n\geq p$ then $\dfrac{a^n}{a^p} = a^{n-p}$; if $n\leq p$ then $\dfrac{a^n}{a^p} = \dfrac{1}{a^{p-n}}$

<u>examples</u> :

1) $\dfrac{c^3}{c^2} = c^{3-2} = c^1 = c$

2) $\dfrac{c^2}{c^3} = \dfrac{1}{c^{3-2}} = \dfrac{1}{c^1} = \dfrac{1}{c}$ in this example, unknown c is written at the bottom, as there was a power of a bigger index at the bottom, and we wanted a positive index in the end.

3) Simplify $\dfrac{6s^7t}{12s^3t^2}$. Working $\dfrac{6}{12}=\dfrac{1}{2}$; $\dfrac{s^7}{s^3}=s^{7-3}=s^4$; $\dfrac{t}{t^2}=\dfrac{1}{t^{2-1}}=\dfrac{1}{t}$ therefore $\dfrac{6s^7t}{12s^3t^2} = \dfrac{s^4}{2t}$

<u>Vocabulary</u>: to evaluate – to substitute ... for ...

Exercise 7 : *Simplify, then read* :

① $\frac{k^2}{k^4}$ ② $-\frac{a^3}{a}$ ③ $-\frac{b^5}{b^5}$ ④ $\frac{s^3}{s^5}$

⑤ $\frac{4c^3}{8c^5}$ ⑥ $-\frac{15a^8}{3a^5}$ ⑦ $\frac{24d^3}{3d^4}$ ⑧ $\frac{14a^7b^2}{21a^3b^3}$

⑨ $\frac{(-2)^3\times 5^4}{2^4\times(-5)^2}$ ⑩ $\frac{0.7\times 10^{-3}\times 10^8}{49\times 10^5}$

VII. Power of a product

$a, b \in \mathbb{R}$ $n, p \in \mathbb{Z}$

$(ab)^n = a^n b^n$: to raise a product to a power, raise every factor to the power

example : Work out $A = (7\times 11)^2\times 11^5\times 7^2$

working $A = 7^2\times 11^2\times 11^5\times 7^2$; $A = 7^2\times 7^2\times 11^2\times 11^5$; $A = 7^{2+2}\times 11^{2+5}$; $A = 7^4\times 11^7$

VIII. Power of a quotient

$a, b \in \mathbb{R}$ $n, p \in \mathbb{Z}$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$: to raise a fraction to a power, raise both top and bottom to the power

example : Work out $A = \left(\frac{7}{11}\right)^2\times\left(\frac{11}{7}\right)^4$

working $A = \frac{7^2}{11^2}\times\frac{11^4}{7^4}$; $A = \frac{7^2\times 11^4}{11^2\times 7^4}$; $A = \frac{11^2}{7^2}$;

IX. Power of a power

$a, b \in \mathbb{R}$ $n, p \in \mathbb{Z}$

$\left(a^n\right)^p = a^{np}$: to raise the power of a number to a power, multiply the indices

example : Simplify $A = \frac{(-15)^3\times(-4)^2}{25^2\times(-6)^2}$

working $(-15)^3 = ((-1)\times 5\times 3)^3 = (-1)^3\times 5^3\times 3^3 = -5^3\times 3^3$; $(-4)^2 = (-1)^2\times\left(2^2\right)^2 = 2^4$; $25^2 = \left(5^2\right)^2 = 5^4$;

$(-6)^2 = 6^2 = (2\times 3)^2 = 2^2\times 3^2$ therefore $A = -\frac{5^3\times 3^3\times 2^4}{5^4\times 3^2\times 2^2} = -\frac{5^3}{5^4}\times\frac{3^3}{3^2}\times\frac{2^4}{2^2} = -\frac{1}{5}\times 3\times 2^2 = -\frac{12}{5}$

Exercise 8 - *Simplify and tell what rules are used* :

① $B = \frac{(-3)^5\times 5^4}{15^2\times 3^4}$ ② $C = \frac{(-35)^2\times(-3)^4}{(-5)^3\times 21^2}$ ③ $D = \frac{\left(\frac{33}{34}\right)^3\times 7^4}{\left(21^2\right)^2\times\left(\frac{17}{11}\right)^5}$