I. What is a surd?

A **surd** /sɜːd/ is a real number that can be expressed only using the **square root sign** $\sqrt{\ }$ (or **radical sign**). It always contains an irrational root, *i.e.* the ________ root of a number which is not the square of an integer.

The number under the radical sign is called the **radicand**.

Examples:

The square root of 2, $\sqrt{2}$, is a surd, as 2 is not the ________ of an integer.

The ________ ______ of 3, $\sqrt{3}$, is a surd also.

$\sqrt{9}$ is not a ______, as $3^2=9$. $\sqrt{9}$ is equal to 3.

$1+\sqrt{7}$ is also a ______. Since it contains a rational and an irrational term, it is called a **mixed surd**.

$\sqrt{13}$ is a surd, it is called a **pure surd** because there is no rational term.

$\sqrt{2}+\sqrt{3}$ is also a ______ ______ because there is no ____________ term.

Exercise 1: *which of the following numbers are surds?*

a) $\sqrt{19}$ b) $\sqrt{169}$ c) $1-\sqrt{49}$ d) $\frac{1+\sqrt{5}}{2}$ e) $\sqrt{16}$ f) π g) $\sqrt{48}$

II. Simplification of surds

If we write a surd involving the square root of a composite number (a number which is not a prime number) such as $\sqrt{60}$, $\sqrt{75}$ or $\sqrt{98}$, we note that the radicand is a multiple of a perfect square. In that case, it is required to simplify and express the number in terms of the **simplest possible surd**:

$\sqrt{60}=\sqrt{4\times 3\times 5}=\sqrt{2^2\times 15}=2\sqrt{15}$. Note that we usually keep $\sqrt{15}$ instead of $\sqrt{3}\sqrt{5}$.

$\sqrt{75} = \sqrt{25\times 3} = \sqrt{5^2\times 3} = 5\sqrt{3}$

$\sqrt{98}=\sqrt{2\times 49}=\sqrt{2\times 7^2}=7\sqrt{2}$

Exercise 2: *express in terms of the simplest possible surd*

a) $\sqrt{8}$ b) $\sqrt{12}$ c) $\sqrt{50}$ d) $\sqrt{18}$ e) $\sqrt{200}$ f) $\sqrt{72}$ g) $\sqrt{125}$

h) $\sqrt{288}$ i) $\sqrt{450}$ j) $\sqrt{2000}$ k) $\sqrt{63}$ l) $\sqrt{112}$ m) $\sqrt{2310}$ n) $\sqrt{484}$

Vocabulary: radical sign – radicand – simplest possible surd – square root sign – (mixed / pure) surd

III. Calculations with surds

Like surds have the same radicand, whereas **unlike surds** have different radicands.

Whenever performing calculations with surds, it is expected to simplify ____ _____ after expressing all terms as the __________ possible surds. Results should be left in surd form unless an approximation is asked for or unless good sense calls for a decimal number with a fixed number of decimal digits such as a price.

Examples:

$3\sqrt{2}$ and $5\sqrt{2}$ are ______ _______ as they have the same radicand, 2.

$\sqrt{2}$ and $2\sqrt{5}$ are ________ _______ since their radicands are different.

Only like surds can be simplified when added or subtracted.

Examples:

$3\sqrt{2} + 5\sqrt{2} = 8\sqrt{2}$

$\sqrt{2} - 2\sqrt{5}$ cannot be simplified further, as the two terms are already the simplest possible surds and they are ________ _______.

Exercise 3 : *simplify*

a) $2\sqrt{18} - \sqrt{50}$ b) $\sqrt{27} - \sqrt{75}$ c) $3\sqrt{28} + 2\sqrt{63}$ d) $5\sqrt{245} - 7\sqrt{125}$

IV. Rationalizing the denominator:

Usually, when surds are involved in a fraction, it is expected to remove surds from the denominator.

This is called **rationalizing** the denominator. /ˈræʃ.ən.əlaɪz/

If the denominator is a single pure surd, multiply both the numerator and the denominator by the square root that appears in the denominator

Example: $\frac{7}{3\sqrt{2}} = \frac{7\sqrt{2}}{3\sqrt{2}\sqrt{2}} = \frac{7\sqrt{2}}{6}$

If there is a mixed surd or a sum or difference of unlike surds, you've got to multiply both the numerator and the denominator by the **conjugate surd** of the denominator as follows:

$$\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{\sqrt{5}-\sqrt{2}}{(\sqrt{5})^2-(\sqrt{2})^2} = \frac{\sqrt{5}-\sqrt{2}}{3}$$

Vocabulary: conjugate surd – like surds – to rationalize – unlike surds