

## 1. Prime numbers

**Definition:** The **factors** (or divisors) of any whole number are the numbers that divide exactly into the given number.

**Example:** 4 is a factor of 12 because 4 divides into 12. The other factors are 1; 2; 3; 6 and 12.

1 is a factor of any natural number. Every number is a factor of itself.

**Definition:** A **prime number** is a whole number greater than 1 that has only two factors, 1 and itself.

The first fifteen prime numbers are 2; 3; 5; 7; 11; 13; 17; 19; 23; 29; 31; 37; 41; 43 and 47.

It is easy to find the first prime numbers with **Eratosthenes' sieve**. /erə'tos.θə'niz sɪv/

1 is not a prime number since it has only one factor, not two different ones.

2 is the first prime number, it is also the only even prime number: "2 is the oddest prime number".

**Composite numbers** are numbers which have more than two factors, such as 6; 10 or 77.

The **prime factorisation** (US : factorization) of a whole number is the representation of that number as a product of factors which are all prime numbers. It is possible to find the prime factors of any given number by successively dividing the number by the prime numbers in increasing order. Start with the smallest number that divides exactly, repeat as many times as necessary with the same prime number, then try the next prime number in the same way, and stop when you are left with 1.

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**Exercise 1 :** write down the following numbers as the product of their prime factors.  
Then find all their different factors.

- a) 24
  - b) 60
  - c) 108
  - d) 126
  - e) 70
  - f) 100
  - g) 65536
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**Vocabulary:** composite numbers – to divide into a number – divisor – factor – prime factorisation – prime number – (Eratosthenes') sieve

## 2. Highest common factor and lowest common multiple

Definition: The **highest common factor (HCF)** of two or more numbers is the largest factor that is common to each of the given numbers.

In other words, the HCF is the largest number that will divide exactly into each of the numbers.

Whenever you \_\_\_\_\_ a fraction to its \_\_\_\_\_, you divide both the \_\_\_\_\_ and the \_\_\_\_\_ by their \_\_\_\_\_.

In order to get the HCF of 2 numbers, you can use **Euclid's algorithm**. /ju.klɪd/

In order to get the HCF of 2 or more numbers, you can express each of the numbers as the product of its prime factors, take all the factors that are common and multiply them together.

Definition: The **least common multiple (LCM)** of two or more numbers is the smallest multiple that is common to each of the given numbers.

In other words, the LCM is the smallest number into which each of the numbers divides exactly.

Whenever you add or subtract fractions, the most convenient common denominator is the LCM.

The product of any 2 numbers is always equal to the product of their HCF and their LCM.

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*Exercise 2 : find the highest common factor in each of the following:*

- |              |                   |                   |                    |
|--------------|-------------------|-------------------|--------------------|
| a) 6 and 10  | b) 16 and 36      | c) 24 and 40      | d) 14 and 35       |
| e) 18 and 63 | f) 11 ; 55 and 99 | g) 27 ; 36 and 63 | h) 45 ; 75 and 105 |

*Exercise 3 : find the lowest common multiple in each of the following:*

- |              |                   |                   |                    |
|--------------|-------------------|-------------------|--------------------|
| a) 6 and 10  | b) 16 and 36      | c) 24 and 40      | d) 14 and 35       |
| e) 18 and 63 | f) 11 ; 55 and 99 | g) 27 ; 36 and 63 | h) 45 ; 75 and 105 |

*Exercise 4 :*

*Prove that the HCF of any two positive integers always divides into their LCM.*

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Vocabulary: (Euclid's) algorithm – HCF – highest common factor – LCM – least common multiple