I What is an equation?

An **equation** is a statement that 2 mathematical expressions (containing at least one **variable**) are **equal**.

If the statement is true only for some values of the **unknowns** (or ________________), it is sometimes called a **conditional equation** : the statement is true under a certain condition. /ɪˈkweɪʒən/

If it is true for all values of the variables, it is called an **identity**. An identity is true whatever the values of the variables.

Examples:

The special expansion $(a+b)^2=a^2+2ab+b^2$ is an __________ as it is true for all values of ***a*** and ***b***.

$(a+b)^2=a^2+b^2$ is a conditional equation, as it is true **if and only if** either ***a*** or ***b*** is equal to zero.

$(u+v)^2=u^2-v^2$ is a __________ __________ , as it is true ___ and ___ ___ $v=0$ or $v=-u$.

$3x+1=x+13$ is a __________ __________ in one unknown, ***x*** , which is the **subject** of the equation.

In practice, we use only the word "equation" to designate a conditional equation.

To **solve** an equation in one unknown means to find the value of the __________ that makes both sides equal in value.

The aim is to end up with 'unknown' = 'number'. Then we say that the equation is solved. A number that makes ________ ________ equal in value is called a **solution** or a **root** of the equation.

Example – Solving an equation

$$4x+3=21-2x$$

$4x+3$ is on the **left hand side** (**LHS**) of the equation, $21-2x$ is on the **right hand side** (**RHS**).

Here is a description of what we do when solving this equation:

- add 2*x* to both sides of the equation $4x+2x+3=21$
- subtract 3 from both sides of the equation $4x+2x=21-3$
- simplify like terms $6x=18$
- divide throughout by 6 $x=3$

In other words, we need to collect all terms in the unknown on one side of the equation, done here by bringing the "2*x*" to the LHS, and all the terms independent of the unknown on the other side.

Always remember that you must do the same thing to both sides of the equation. Any addition, or subtraction is allowed, as well as multiplications or divisions by non zero numbers.

It is **trickier** to square both sides of the equation, since two numbers the squares of which are equal can be equal or can be opposite (same absolute value but unlike signs).

It is a good practice to check your ___________ after you have solved an ___________ by **substituting** the solution **for** the ___________ in the equation you had at the beginning.

Vocabulary: conditional – equal – equation – identity – if and only if – left hand side – LHS – RHS – right hand side – root – solution – to solve – subject – to substitute ... for ... – tricky – unknown – variable

II Simultaneous linear equations in two unknowns

An equation such as $5x+3y=4$ is called a **linear equation in two unknowns**, x and y.

Simultaneous ________ ___________ are a pair of such equations.

Solving a pair of _______________ equations requires finding the values of the two variables that make both equations true at the same time.

As in the case of a single equation in one unknown, when you solve a pair of simultaneous equations, it is a good idea to check your answer by substituting the solutions for the unknowns in the two original equations.

Simultaneous linear equations are solved with the following steps:

- Write both equations in the form $ax+by=c$ and number the equations (I) and (II).
- Multiply one or both of the equations by a number in order to make the coefficients of x or y the same in the two equations, ignoring signs.
- Keep one of the equations as it is and replace the other one by the sum or the difference of the two equations, so as to remove the variable with equal coefficients. You now have a pair of simultaneous linear equations, one in 2 unknowns, the other one in 1 unknown.
- Solve the equation in 1 unknown and substitute the solution for the unknown in the other equation.
- Solve the remaining equation in one unknown.

Example: solve the following simultaneous equations

$$\begin{cases} 5x-2y=7 & \text{(I)} \\ 4x+3y=24 & \text{(II)} \end{cases}$$

$$\begin{cases} 15x-6y=21 & \text{(I)}\times 3 \\ 8x+6y=48 & \text{(II)}\times 2 \end{cases}$$

Multiply throughout the first equation by 3 and the second one by 2

$$\begin{cases} 5x-2y=7 & \text{(I)} \\ 23x\quad=69 & \text{(I)}\times 3+\text{(II)}\times 2 \end{cases}$$

Replace the second equation by the sum of the two equations

$$\begin{cases} 5\times 3-2y=7 \\ x\quad=3 \end{cases}$$

Solve the equation in one variable and substitute in the other one

$$\begin{cases} y=4 \\ x=3 \end{cases}$$

Solve the remaining equation in one variable

Note that in the third step, it is better to take back the original form of the first equation in order to make the final calculations with smaller numbers.

Normally, simultaneous linear equations can have a single solution, no solution at all or **infinitely many** solutions.

The last case occurs whenever the second equation is obtained by multiplying the first one throughout by a constant.

Vocabulary: equation in n unknown(s) – infinitely many – linear equation – to multiply throughout – simultaneous

III Using equations to solve problems

A numerical problem given in words can often be translated into an equation or a pair of simultaneous equations. The solution of the equations will give the solution to the problem. After solving the equations, you should not forget to do two things:

- **Check** that your answer makes sense in real life. This is an easy way to find some mistakes.
- If the answer seems to make sense, test your solution in the problem itself, not in the equation that you have written, since the equation itself might be wrong.

Example:

Ann has a number of candies. Her friend Ben has 15 less than she has. Between them, they have 41 candies. How many candies each have Ann and Ben?

Solution: you must write a few sentences so that your solution is understandable.

Let x be the number of candies Ann has. Since Ben has 15 less than Ann, he has $(x-15)$ candies.

Then, as they have 41 candies together, $x+(x-15)=41$.

Therefore, $2x-15=41$, so $2x=56$ which means that x is equal to 28.

So, Ann has 28 candies, and Ben has 13 candies. Their numbers of candies indeed add up to 41.

Note that the sentences you write are as important as the actual solving of the equations. In order to make your life easier, you can use whatever letters are convenient to designate the unknowns, as long as you don't use the same letter to designate two different things.

Example:

A concert was held in a hall that can hold 160 people. There are two prices of tickets, 5€ and 3€. One evening when the hall was full, six hundred euros were collected. Find how many of each ticket was sold that day.

Solution:

Let f be the number of full price tickets and r the number of reduced price tickets that were sold.

Since the hall was full, the numbers of tickets sold add up to 160, so $f+r=160$.

The amount of money collected was 600 €. Thus, $5f+3r=600$.

$$\begin{cases} f+r=160 \\ 5f+3r=600 \end{cases} \Leftrightarrow \begin{cases} 3f+3r=480 \\ 5f+3r=600 \end{cases} \Leftrightarrow \begin{cases} f+r=160 \\ 2f=120 \end{cases} \Leftrightarrow \begin{cases} 60+r=160 \\ f=60 \end{cases} \Leftrightarrow \begin{cases} r=100 \\ f=60 \end{cases}$$

Therefore, 100 reduced price and 60 full price tickets were sold.

Checking the answers:

$100 + 60 = 160$ $3 \times 100 + 5 \times 60 = 600$

Checking that way will detect a mistake such as exchanging the prices in the second equation.

Vocabulary: to check

IV Exercises

<u>Exercise 1</u> : Nine children share €53.10 *equally between them. How much will each one receive?*

<u>Exercise 2</u> : Three consecutive odd numbers add-up to 51. What are those numbers?

<u>Exercise 3</u> : Mr. Wilson regularly attends football matches when his team plays at home.

*For **x** games, he buys a seat in the stands. Each seat costs* £9. *Write down an expression for the cost of these **x** games.*

For the remaining games, he buys a ticket in the terraces. Each of those tickets cost £5. *If he attends* 24 *games in a season, write down an expression for the cost of the tickets in the terraces.*

The cost for the whole season was £176. *How many matches did Mr. Wilson watch from the stands? Check your answer.*

<u>Exercise 4</u> : Seven books and three magazines cost €82. Two books and one magazine cost €24. Work out the price of one book and the price of one magazine. Check your answer.

How much do nine books and four magazines cost?

<u>Exercise 5</u> : Ms Mallone bought eggs in the grocery store this morning. She used one quarter of the eggs in order to prepare scrambled eggs for lunch. Then she used half the remaining ones to bake a cake in the afternoon. Finally, she hard boiled one third of the eggs left for tomorrow's lunch, and she is left with 6 eggs only. How many eggs did she purchase in the morning?

<u>Exercise 6</u> : After disposing of the previous owners, the men aboard a pirate ship share booty they obtained. The pirate captain's share is twice the share each of his two lieutenants gets. In turn, each lieutenant gets five times as much as any sailor. Knowing that there are 52 sailors and that the booty consists in 2952 gold pieces, how much will each of the pirates get?

<u>Exercise 7</u> : In the grocery store at the corner, a bottle of orange juice costs 10 cents more than a bar of chocolate. Two bars of chocolate and one bottle of orange juice cost €2.50. What is the cost of a bar of chocolate and what is the cost of a bottle of orange juice?

<u>Exercise 8</u> : A high speed train requires three hours to drive the 600 km from Paris to Bordeaux. It first drives one hour on the high speed track, then two hours on the regular track. Its speed on the high speed track is twice its speed on the regular track. What is its speed on the two different parts of the trip?

<u>Exercise 9</u> : A woman's age this year is four times the age of her son. In five years' time she will be three times as old as her son. What age is each of them now?

<u>Exercise 10</u> : Two cylindrical buckets hold 18 and 6 liters of water, respectively. To each bucket is now added the same amount of water, so that the first now holds twice as much as the second one. How much water was added in each of the buckets?

<u>Exercise 11</u> : The Dublin zoo purchases 5 zebras and 3 yaks for €1,450. The Cork zoo purchases 4 zebras and 5 yaks for €1,550. What is the price of a yak, what is the price of a zebra?

<u>Exercise 12</u> : John usually rides his bicycle when going to school. Today, instead of cycling, he walked. His speed when walking is 18 km/h less than his speed when riding his bike, and it took half an hour longer than usual, which is to say four times as long as usual. How fast does John walk, how fast does he ride his bike, and how far is his home from his school?