I Definition and vocabulary

The sign ' % ' stands for **percent** /pəˈsent/ (or per cent), which means "out of one hundred".

45% is a **percentage** /pəˈsen.tɪdʒ/**.** It means 45 out of one hundred.

Note that when using percentages, one should always be aware that "per cent *of a certain quantity*" is implied when it is not explicitly stated.

Examples:

- 52% *of the newborn children* are boys.
- 16.2% *of the French people* did not vote on April 22nd, 2007.
- During the sales, the prices of all the clothes are reduced by 30% (*of the original price*)
- The interest rate on a savings account is 3% (*of the amount on which interest is paid*).

Note that 50% means one half, 25% means one fourth and one third is 33% or 33.3% or 33.33% depending on the choice made for rounding off.

II Changing numbers to percentages / Working out percentages

Rule: Multiplying by 100 %

To find what percentage number $\boldsymbol{a}$ is of number $\boldsymbol{b}$, multiply the fraction $\frac{\boldsymbol{a}}{\boldsymbol{b}}$ by 100%.

Examples:

Sam got 40 out of 50 in his math test. What is his mark as a percentage?

$$\frac{\mathbf{40}}{\mathbf{50}} \times \mathbf{100}\,\% \;=\; \frac{\mathbf{40} \times \mathbf{100}}{\mathbf{50}}\,\% \;=\; \mathbf{80}\,\%$$

The Spitfire's nominal speed is 602 km/h. If the climbing speed of a Spitfire is 486 km/h, work out the ascension rate as a percentage of the nominal speed.

$$\frac{\mathbf{486}}{\mathbf{602}} \times \mathbf{100}\,\% \;=\; \frac{\mathbf{486} \times \mathbf{100}}{\mathbf{602}}\,\% \;=\; \mathbf{80.\,73}\,\%$$, correct to 2 d.p.

Exercise 1:

1. *6 students out of a class of 30 are studying Japanese. What percentage of the class is the size of the group of students who do not study Japanese?*

2. *The Spitfire's top diving speed is 698 km/h, its top nominal speed is still 602 km/h. Express its top diving speed as a percentage of its nominal speed.*

Vocabulary: percentage – percent (per cent)

III Changing percentages to fractions or decimals

Rule : To change a percentage into a fraction :
Changing percentage to fraction amounts to removing the % sign and writing the number as a fraction with 100 on the bottom.

Don't forget to _____ _____ the fraction to its _____ _____ .

Rule : To change a percentage into a decimal :
Changing percentage to decimal amounts to removing the % sign and dividing the number by 100.

Example: "You need 90% of the time to complete the last 10% of the job" (Engineering saying)

$$90\,\% = \frac{90}{100} = \frac{9}{10} \qquad 10\,\% = \frac{10}{100} = \frac{1}{10}$$

So you can state: "You need _____ _____ of the time to complete the last _____ of the job" instead.

Exercise 2: Convert the percentages into fractions and rewrite the sentences :

At least 50% of the French population has been living in cities since 1921.
85% of the French electricity is produced in nuclear power plants.
33.3% of the Americans are overweight.

IV Increasing or decreasing a number by a percentage

Example: In January, 2007, a jar of cranberry preserve cost £ 2.50 at the grocery shop around the corner. In February, the price had increased by 8%. Calculate the price in February.

Solution 1:

- Calculate the amount by which the price has increased: $2.50 \times \frac{8}{100} = 0.20$ (8% of £ 2.50 is £ 0.20)
- Then _____ that amount ___ the original price: 2.50 + 0.20 = 2.70
- Don't forget to conclude with a correct sentence: "The price in February was £ 2.70"

Solution 2 (shortcut for solution 1) :

- First calculate the final price as a _____ of the initial price: 100% + 8% = 108%
- _____ the initial price __ that percentage: 108% of 2.50 equals $2.50 \times \frac{108}{100}$ (which equals 2.70)
- Conclude with a correct sentence: "The price in February was £ 2.70"

Solution 3 (shortcut for solution 2) :

- Directly multiply the price by $\left(1 + \frac{x}{100}\right)$ written as a decimal number if it increases by x %.
  Here, we get 2.50 × 1.08 = 2.70 so the final price is € 2.70.

Vocabulary: to increase (or decrease) by a percentage

Example: In April, 2006, the new video game "Master of hack and slash" was sold for £ 59.99 in every computer games shop. One year later, since the brand new "Grand Master of hack and slash" came out, the previous game's price was reduced by 40%. Calculate its new price.

Solution 1:

- Calculate the amount by which the price has _____: $\mathbf{59.99 \times \frac{40}{100} = 23.996}$ (rounded off to **24**)
- Then _____ that amount ____ the original price : **59.99 – 24 = 35.99**
- Don't forget to conclude with a correct sentence: "The new price is £ 35.99"

Solution 2 (shortcut for solution 1):

- First calculate the _____ price as a _____ of the initial price: 100% – 40% = 60%
- Multiply the _____ price by that percentage: 60% of 59.99 equals 35.994, rounded off to 35.99
- Conclude with a sentence: "The new price of the video game is £35.99"

Solution 3 (shortcut for solution 2):

- Directly multiply the price by $\left(\mathbf{1-\frac{x}{100}}\right)$ written as a decimal number if it decreases by $\boldsymbol{x}$ %. Here, we get $\mathbf{59.99 \times 0.6 = 35.994}$ so the final price is € 35.99.

V Find the initial value knowing the increased or decreased one

Example: The price of a bottle of perfume at London Heathrow airport in the duty free zone is £84, including the **Value Added Tax** (**VAT**) at 19.6%. If an Australian customer buys the perfume before flying back home to Brisbane, he will not have to pay the tax. How much will the perfume cost him?

Since we don't know the price before applying the 19.6% tax, we have to solve an equation.

Let $\boldsymbol{x}$ be the price before tax. Since the tax rate is 19.6%, the price after tax is equal to $\boldsymbol{x}$ plus 19.6% of $\boldsymbol{x}$, *i.e.* 1.196 $\boldsymbol{x}$ (see section IV if you fail to understand how we get to that).

Hence, $\mathbf{1.196\,x = 84}$; $\mathbf{x = \frac{84}{1.196}}$; $\mathbf{x = 70.23}$ correct to 2 decimal places

So, the Australian customer will pay only £ 70.23 for the bottle of perfume.

Exercise 3:

1. *The price of a car is £ 9,000 including VAT at 25%. Find the price of the car before the VAT is added.*
2. *After an increase of 20 %, a motorbike was sold for £ 2,640. What was the original price ?*

Vocabulary: Value Added Tax (VAT)

Example: During the sales, a leather jacket is sold at £ 165 with a label "30 % off: the price already includes the discount". What was the price before the sales?

Since we don't know the price before applying the 30% discount, we'll solve an equation.
Let x be the price before the discount (the original price).
Since the discount is 30% of the price *before the discount*, we pay only 70% of the original price, so 70% of x equals £ 165.

$$0.7\,x = 165 \quad ; \quad x = \frac{165}{0.7} \quad ; \quad x = 235.71$$, correct to 2 _____ _____.

The original price was £ 235.71

VI Proportional parts

Rule : To divide a quantity into **proportional parts** , write each part as a fraction of the quantity.

Example : Divide a 80 cm long stick into two parts **in the ratio** 7 : 3 (*reads "seven to three"*)
1- count the number of portions, by adding the numbers in the ratio : $7+3 = 10$

2- write each part as a fraction of the number of portions :

1st part : 7 portions out of 10, hence $\frac{7}{10}$

2nd part : 3 portions out of 10, hence $\frac{3}{10}$

3- multiply the quantity by the fractions to find the parts :

1st part : $\frac{7}{10} \times 80 = 56$ cm

2nd part : $\frac{3}{10} \times 80 = 24$ cm

Note : A quantity can be divided in more than 2 parts, for instance, divide Q in the ratio $a : b : c$.
You get three parts, computed as $\frac{a}{a+b+c}Q$, $\frac{b}{a+b+c}Q$ and $\frac{c}{a+b+c}Q$

Exercise 4 :

1. *A bar of chocolate has 24 squares. If the bar is divided between three children in the ratio 3 : 2 : 1, how many squares does each get ?*
2. *£ 60 is divided between A and B, so that A gets twice as much as B. How much does each get ?*
3. *A piece of wire 72 cm long is to be bent into the shape of a triangle with sides in the ratio 5 : 4 : 3. Is the triangle right-angled? What are the lengths of the sides ?*
4. *At a football match, the ratio of the home supporters to away supporters was 5 : 3. 15,000 away supporters attended the match. How many home supporters were at the match ?*

Vocabulary: proportional parts – in the ratio n to p