I. Symbols

$>$	**is greater than**	$\geq$	**is greater than or equal to** ($\geqslant$)
$<$	**is less than**	$\leq$	**is less than or equal to** ($\leqslant$)

If $a > b$, then $a - b > 0$: if ***a*** is greater than ***b***, then ***a*** minus ***b*** is **positive**
If $a \leq b$, then $a - b \leq 0$: if ***a*** is less than or equal to ***b***, then ***a*** minus ***b*** is **negative or zero**

If $a \geq 0$, ***a*** is **non-negative**. If $a \leq 0$, ***a*** is **non-positive**.

Exercises

1) Complete and write the sentences :

a) If $a < b$, then $a - b$ ______ : ______________________________

b) If , $a \geq b$ *then* ____________ *:* ______________________________

2) Make sentences using expressions like « is greater than » or « is less than » that would define :

a) an acute angle in relation to 0° and 90° :

b) an obtuse angle in relation to 90° and 180° :

c) the hypotenuse of a right-angled triangle in relation to both other sides :

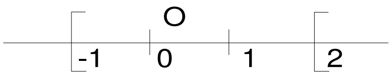
II. Intervals or Ranges

- $a < x < b$: *x* **lies between** *a* **and** *b* (or ***x*** is greater than ***a*** and less than ***b***)
i.e. $x \in]a;b[$ or the English way $x \in (a;b)$: ***x*** belongs to the **open interval** or **range** $]a;b[$

- $a \leq x \leq b$: *x* lies between ***a*** and ***b*** (or ***x*** is greater than or equal to ***a*** and less than or equal to ***b***)
i.e. $x \in [a;b]$: ***x*** belongs to the **closed** interval or range $[a;b]$ /reɪndʒ/

- $a < x \leq b$: ***x*** lies between ***a*** and ***b*** , **excluding *a*** and **including *b*** (or ***x*** is greater than ***a*** and less than or equal to ***b***) *i.e.* $x \in]a;b]$: ***x*** lies within the range $]a;b]$ or $(a;b]$, **open at *a*** and **closed at *b***

- ***a*** and ***b*** are the **end points** of the interval



- A range, like $[-1;2[$, can be represented on a **number line** (line on which each point represents a real number)

Exercises

3) For each interval, write two sentences as before and represent it on a number line (be careful with the choice of the unit) :

a) $-2 < x < 3$ *b)* $0 \leqslant x \leqslant \frac{3}{2}$ *c)* $-\frac{1}{3} < x \leqslant \frac{2}{3}$

Vocabulary: closed (at) – end points – excluding – greater than (or equal to) – including – interval – less than (or equal to) – to lie between ... and ... – negative (positive) (or zero) – non-negative – non-positive – number line – open (at) – range

III. Solving inequalities

An **inequality** remains true when the same number is added to (or subtracted from) both sides or when both sides are multiplied (or divided) by the same positive number.

BUT

An inequality (or its **direction**, or its **order**) is **reversed** if you multiply (or ___________) _________ __________ by a ____________ number.

The two expressions « $x > y$ » and « $x < y$ » are said to have **opposite senses**.

Exercises

4) *State the range of values of* **x** *for which each of the following inequalities is true and illustrate this range on a number line - remember you can collect the letter terms on both LHS (Left Hand Side) and RHS (R________ H_________ S_________) :*

   a) $5-2(x-8) < 7x-3(x-2)$

   b) $\frac{3}{2}x-\frac{1}{4} \geq 1-\frac{1}{2}(7-9x)$

   c) $\frac{x}{9}-\frac{x-1}{3} \leq 1-\frac{2(x+1)}{3}$

IV. Conditional and unconditional inequalities

As with equations, inequalities can be **conditional** or **unconditional**.

An unconditional inequality is one that holds for all values of the variables, *i. e.* it is the analogue of the identity in equations.

An example would be : $x^2 \geq 0$ which is true for all values of x.

A conditional inequality is true only for certain values of the variables, for instance $2x+1>11$ is true only for $x>5$

Inequalities involve **transitive** relationships. Thus, if $a > b$ and $b > c$, it follows that $a > c$.

Exercises

5) *Prove* $a^2+b^2 \geqslant 2ab$ *for all real values of* **a** *and* **b**.
   Hence, prove that $a^2+b^2+c^2 \geqslant ab+bc+ca$ *for all real values of* **a** , **b** *and* **c**.

6) *If* **p**, **q** *> 0, prove :*

   a) $p+\frac{1}{p} \geqslant 2$　　b) $\frac{p}{q}+\frac{q}{p} \geqslant 2$　　c) $(p+q)\left(\frac{1}{p}+\frac{1}{q}\right) \geqslant 4$

Vocabulary: (un)conditional inequality – direction – opposite senses – order – to reverse – transitive