There are many types of **averages** /ˈæv.ᵊr.ɪdʒ/ . Two that we will meet are called the **mean** and the **median**. They are also known as measures of **central tendency**.

1 Mean

The arithmetic mean, in short the mean, is the proper name for what most people call the average. /miːn/

Definition : The mean of a set of values is the sum of all the values divided by the number of values. The formula is written as $\bar{x} = \frac{\sum_{i=1}^{i=n} x_i}{n}$, where n is the number of values and $(x_i)_{1 \le i \le n}$ are the values,

in short : $\bar{x} = \frac{\sum x}{n}$ *read* : « x bar is equal to sigma (the sum) of all the x-values divided by n »

2 Median

The values are arranged in ascending, or descending, order of size, then the median is a number dividing the set in two subsets of equal size : half the values are less than this median, half the values lie above.

If the number of values is even, then the median is the mean of the two middle values.

If the number of values is odd, then the median is the middle value. /ˈmiː.dɪ.ən/

Example : Let {3 ; 7 ; 10 ; 5 ; 4 ; 2 ; 2 ; 8 ; 4 }be an array of numbers. Find the mean and median.

Mean : $\bar{x} = \frac{\sum x}{9} = \frac{3+7+10+5+4+2+2+8+4}{9} = \frac{45}{9} = 5$

Median : first step, write in ascending order 2 ; 2 ; 3 ; 4 ; 4 ; 5 ; 7 ; 8 ; 10
there are 9 values, that is an odd number of values, the median is the middle value : 4

3 Frequency distribution

If the values in a distribution are arranged in ascending or descending order, showing their corresponding frequencies, the distribution is called a **frequency distribution**. /ˈfriː.kwənt.sɪ/

A table showing such a set of data is called a **frequency distribution table**.

3.1 Mean

To find the mean of a frequency distribution, do the following :

- ✔ Multiply each value by its corresponding frequency;
- ✔ Sum all these products;
- ✔ Divide this sum by the sum of all the frequencies.

That is $\bar{x} = \frac{\sum_{i=1}^{i=n} f_i x_i}{\sum_{i=1}^{i=n} f_i}$ where :

n is the number of measurements,

$(x_i)_{1 \le i \le n}$ are the values of the measurements,

f_i is the frequency of the measurement x_i ,

in short : $\bar{x} = \frac{\sum f x}{\sum f}$

Vocabulary average – frequency distribution (table) – mean – median – central tendency

3.2 Median

As the values are arranged in order of size in a frequency distribution table, the median can be read by looking at the row displaying the frequencies :

- ✔ add up all the frequencies : this is the total number of values in the set;
  - o if it is odd, add 1 and divide by 2 for computing the position of the median in the ordered set;
  - o if it is even, divide by 2, save and add 1 for computing the positions of the two values which mean is the median
- ✔ add up the frequencies from the left until you reach the said position or pair of positions;
- ✔ retrieve the corresponding value, or pair of values;
- ✔ work out the median : the value itself if it is single, the mean if there is a pair of values.

Example : A test consisted of five questions. 1 mark was awarded per question for a correct solution and no marks for an incorrect solution. The following frequency distribution table shows how a class of students scored in the test. Calculate the mean and median marks.

mark	0	1	2	3	4	5
Number of students	1	3	6	7	7	4

Mean : $\bar{x} = \frac{\sum fx}{\sum f} = \frac{\mathbf{1(0)+3(1)+6(2)+7(3)+7(4)+4(5)}}{\mathbf{1+3+6+7+7+4}} = \frac{\mathbf{0+3+12+21+28+20}}{\mathbf{28}} = \frac{\mathbf{84}}{\mathbf{28}} = \mathbf{3}$

Median : The sum of all the frequencies is $\sum f = \mathbf{28}$

This sum is even, then we compute two positions : $\frac{\mathbf{28}}{\mathbf{2}} = \mathbf{14}$ and $\mathbf{14+1} = \mathbf{15}$

We are looking for the 14th and the 15th marks, taken one by one. We work out a table showing the partial sums of frequencies :

Marks	0	1	2	3	4	5
Number of students scoring the mark	1	3	6	7	7	4
Number of students scoring **up to** the mark	1	1+3 = 4	4+6 = 10	10+7 = 17	17+7 = 24	24+4 = 28

Both 14th and 15th students scored 3, hence the median $\frac{\mathbf{3+3}}{\mathbf{2}} = \mathbf{3}$

3.3 Grouped frequency distribution

Sometimes the range of the values is very wide and it is not suitable to show all the values individually. When this happens we arrange the values into suitable groups called **class intervals**. When the information is arranged in class intervals, it is not possible to calculate the exact value of the mean. However it is possible to estimate it by using the **mid-interval value** of each class interval. The easiest way to find the mid-interval value is to add the two extreme values and divide by 2.

example :

3.4 Weighted mean

A weighted mean is one where the frequencies are replaced by **weights.** The weight /weɪt/ is a measure of importance of a particular value, *i.e.* each value has a statistical weight attached to it.

Calculating a weighted mean is exactly the same as finding the mean of a frequency distribution, except that the frequencies are replaced by weights, in short :

$$\bar{x} = \frac{\sum wx}{\sum w} \quad \text{where } (w_i)_{1\le i\le n} \text{ are the weights.}$$

Vocabulary class interval – histogram – mid-interval value – (frequency) polygon – relative frequency – weight

4 Graphical displays

4.1 Bar graph

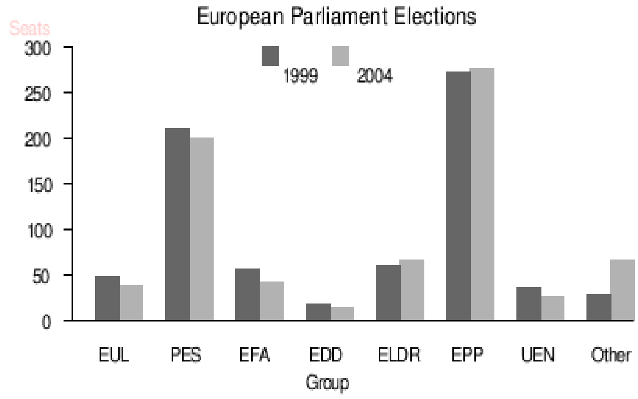
An alternative to the frequency polygon is the **bar graph** , where **non-contiguous** /kənˈtɪg.ju.əs/ bars (or thin rectangles) are drawn, with lengths proportional to the values that they represent. The bars can also be plotted horizontally.

This should mainly be used with **qualitative** or **discrete** /dɪˈskriːt/ data.

Example :

The following graph displays the number of seats allocated to each party group in European elections in 1999 and 2004. The results of 1999 have been multiplied by 1.16933, to compensate for the change in number of seats between those years. Sometimes it can be horizontal.

This bar chart shows both the results of 2004, and those of 1999:



4.2 Frequency polygon

The **frequency polygon** is the simplest graphical display of a frequency table.

The scores are shown on the ***x***-axis and the frequencies are shown on the ***y***-axis : points which cartesian coordinates are these numbers are plotted in a cartesian system of reference. The points are then connected so that together with the ***x***-axis they form a polygon. The points of a frequency polygon corresponding to the lowest and highest values always lie on the ***x***-axis.

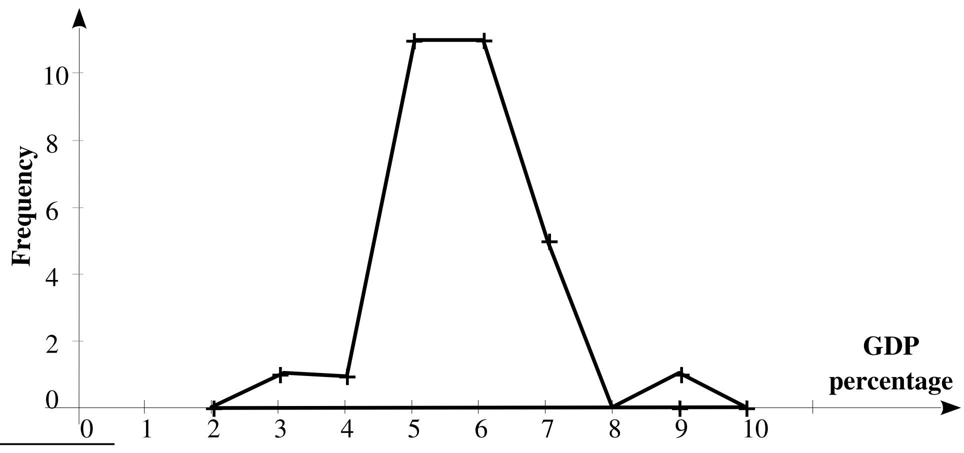
Absolute or relative frequencies may be used. The graph will have the same shape in either case. Frequency is commonly used when the data set is small and relative frequency is used when the data set is large, or when we need to compare several distributions.

Frequency polygons are useful for comparing distributions and are also often used with cumulative frequencies.

Example : Each country spends a certain percentage of its **Gross Domestic Product** on education. The table below shows, among 30 countries all over the world, how many spend a certain percentage. Values have been rounded to the nearest integer. For example, only one country spends 4 % of its GDP on education, while 5 countries spend approximately 7 %.

Percentage of the GDP	3	4	5	6	7	9
Number of countries	1	1	11	11	5	1

These data can be graphicaly represented as a frequency polygon.



Vocabulary bar graph – discrete – frequency polygon – Gross Domestic Product – GDP – non-contiguous – qualitative

4.3 Pie chart

A **pie chart** (or a circle graph) is a circular chart divided into sectors, illustrating relative magnitudes or frequencies or percents. In a pie chart, the arc length of each sector (and consequently its area, and its central angle) is proportional to the quantity it represents.

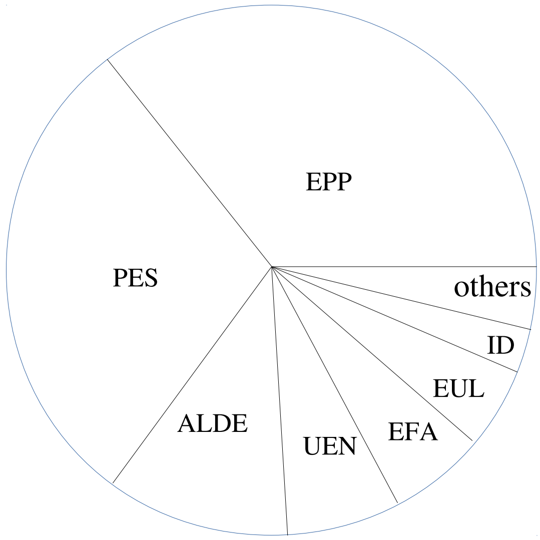
Together, the sectors create a full disk. It is named for its resemblance to a pie which has been sliced.

Example :

The following example chart is based on the results of the election for the European Parliament in 2009.

The following table lists the number of seats allocated to each party group, along with the derived percentage of the total that they each make up. The values in the last column are the derived central angles of each sector, an angle is the product of the percentage by 360°.

Group	Seats	Percent (%)	Angle (°)
EPP	284	36.2	130
PES	215	27.4	100
ALDE	103	13.1	47
UEN	44	5.6	20
EFA	42	5.4	19
EUL	41	5.2	18
ID	24	3.1	11
others	32	4.1	15
Total	785	100	360



The size of each central angle is proportional to the size of the corresponding quantity : the number of seats. Since the sum of the central angles has to be 360°, the central angle for a quantity that is the fraction Q of the total is $360\,Q$ degrees. In the example, the central angle for the largest group (European People's Party, EPP) is 130° because $\mathbf{0.362 \times 360 \approx 130}$, rounded to the nearest integer.

Exercise 1

The pupils in a certain class were asked how they travelled to school. The results are shown in the table below :

Mode of transport	train	Bus	Bycicle	walking
Number of pupils	4	6	5	9

i. *Show the data in a bar graph*

ii. *Show the data in a pie chart*

Vocabulary pie chart