

I. Definitions

Probability involves the study of the laws of chance : that is associating values to the **chance**, or likelihood, of something happening.

Examples : We may toss coins, cast dice, spin **spinners**, draw cards, etc...

If we carry out an operation, or **experiment**, each completion is called a **trial**.

All possible results that can occur from a trial are called **outcomes**, the whole set of outcomes is the **sample space**.

An **event** is a subset of outcomes, that are selected because they are of a particular interest : in that perspective, these outcomes are said to be **successful**. When a trial produces a successful outcome, we say that the event **happens**, or **occurs**.

Example: We draw a card from a 32 deck, and we are interested in clubs. There are 32 _____, and the successful outcomes that constitute this event are : 7♣ , 8♣ , 9♣ , 10♣ , J♣ , Q♣ , K♣ or A♣.
(hearts _____ - spades _____ - clubs _____ - diamonds _____)

II. Equally likely outcomes

If E is an event, then $P(E)$ stands for the probability that this event occurs.

Definition - WHEN ALL OUTCOMES ARE EQUALLY LIKELY

The measure of the probability of an event E is given by :

$$P(E) = \frac{\text{number of successful outcomes}}{\text{number of possible outcomes}}$$

Since the number of successful outcomes ranges from 0 to the number of outcomes in the sample space, the probability of an event is such that :

$$0 \leq P(E) \leq 1$$

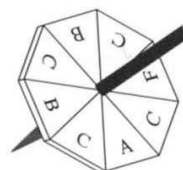
Note : $P(E) = 0$ means that event E is **impossible**

$P(E) = 1$ means that event E is **certain**

Example : what would an example of an impossible event be ? Of a certain event ?

Exercises

- A box contains 36 coloured balls. 12 are red, 15 are blue, 3 are yellow and the rest are white. One ball is selected at random from the box. Calculate the probability of selecting a :
(i) red ball (ii) blue ball (iii) yellow ball (iv) white ball
- A **fair** spinner has eight sides labeled A, B, B, C, C, C, C and F. It is spun once.
What is the probability that the spinner lands on :
(i) A (ii) B (iii) C



Vocabulary certain – chance – event – equally likely – fair – to happen – impossible – likelihood – to occur – outcome – probability – sample space – spinner – successful – trial

III. Combining two events

There are situations where outcomes are quite naturally separated in two groups. In these situations it is often simple to organise the sample space in a **two-way table**.

Example: A fair coin is tossed, and an **unbiased** /ʌn'baɪəst/ 6-sided die is cast. We represent each situation with two-way table for a sample space diagram.

COIN	Tails	T - 1	T - 2	...			
	Heads	H - 1	⊗		⊗		⊗
		1	2	3	4	5	6
DIE							

We use this table for calculating the probability of obtaining a heads and an even number. The total number of outcomes is 12. There are 3 successful outcomes, marked ⊗.

Hence the probability $P = \frac{3}{12} = 0,25$

Exercises

3. In the example above, calculate the probability of obtaining « a tails or a number less than 3 ».
4. Two dice are thrown, one is six-sided and the other one is four-sided.
 - a) How many outcomes are possible ?
 - b) If the scores are added together, calculate the probability that the sum of the scores is :
 - (i) less than 6
 - (ii) 7
 - (iii) greater than 8
 - (iv) nought

IV. Addition rule (∪)

OR RULE :

Let **A** and **B** be two events in the same sample space.

The probability that A or B can happen (*i.e.* that either one of them or both can happen) is given by :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

note : subtracting $P(A \cap B)$ accounts for removing double counting.

Exercise

5. A bag contains three blue discs, five white discs and four red discs. A disc is chosen at random. Find the probability that the disc chosen is :
 - (i) red
 - (ii) blue or red
 - (iii) red or white
6. An unbiased 6-sided die is thrown. Find the probability that the number obtained is :
 - (i) even
 - (ii) prime
 - (iii) even or prime

V. Event not happening (not)

The probability of an event **E** not happening, noted « **not E** », is given by :

$$P(\text{not } E) = 1 - P(E)$$

It can be defined as the **complement** of the event **E**, *i.e.* the collection of all outcomes in the sample space that are not in **E**. An event and its complement cannot occur at the same time.

Exercise

7. The probability that a student will hit the target with a single shot at a rifle range is $\frac{3}{5}$. If he fires one shot, find the probability that he will miss the target.

Vocabulary complement – two-way table – unbiased