I. Mutually exclusive events

Mutually exclusive events are events that cannot both occur in the one experiment.

For instance, if ***E*** is an event, then ***E*** and its complement ***not E*** are mutually exclusive, but events alternately related can be found to be mutually exclusive (mutually exclusive events are not necessarily complementary).

Example : We cast a 10 sided-die.
Event ***A*** is defined as « we cast a number less than 3 » and event ***B*** is « we cast a number greater than 7 ».
A and ***B*** are mutually exclusive, since one experiment produces but one number, that cannot be less than 3 and greater than 7 at the same time.

In that situation, the addition rule is stated in a simpler way :

$$P(A \cup B) = P(A) + P(B)$$

because $P(A \cap B) = 0$

II. Successive events

MULTIPLICATION RULE :
The probability that two events ***A*** and ***B*** both happen, and in that order, is given by :

$$P(A \text{ and } B) = P(A) \times P(B)$$

where $P(B)$ has been worked out assuming that ***A*** has already occured.

note : **successive** events means that order must be taken into account. Also, be very careful where the outcome at one stage does not affect the outcome at the next stage.

Considering an issue in that way helps reduce the need to make out a sample space diagram.

Example :
An unbiased die is cast and a fair coin is tossed.
Find the probability of getting a 5 and a heads.
Solution : $P(5)$ stands for $P(\text{"casting a 5"})$; $P(H)$ stands for $P(\text{"tossing a heads"})$

$$P(5) = \frac{1}{6} \qquad P(H) = \frac{1}{2}$$

$$P(\text{"5 and H"}) = \frac{1}{6} \times \frac{1}{2}$$

It is not necessary to draw a sample space diagram like a tree or a two-way table.

Exercises

1. *Aideen and Bernadette celebrate their birthdays in a particular week (Monday to Sunday inclusive) Assuming that the birthdays are equally likely to fall on any day of the week, what is the probability that :*
   both people were born on a Wednesday?
   one was born on a Monday and the other was born on a Friday ?
   (a) Method 1 – represent the situation with a sample space diagram
   (b) Method 2 – Use the rules of probability : the addition rule and the multiplication rule for events not affecting each other.

Vocabulary – mutually exclusive – successive

2. *A bag contains 6 red and 2 yellow counters.*
   (a) *A counter is drawn at random from the bag and replaced. Then a second counter is drawn at random. Find the probability that :*
      i. *the first is yellow*
      ii. *the first is red and the second is yellow*
      iii. *one is red and the other is yellow*
      iv. *both are the same color*
   (b) *If the first counter drawn is not replaced, find the probability that :*
      i. *the first is yellow and the second is red*
      ii. *one is yellow and the second is red, in any order*
      iii. *both are the same color*

3. *A bag contains 5 black beads and 3 yellow beads. A bead is chosen at random from the bag and not replaced. A second bead is then chosen from the bag.*
   Find the probability that :
   (a) *both beads are black*
   (b) *both beads are yellow*
   (c) *the first bead is yellow and the second bead is black*
   (d) *both beads are of different colors*

4. *Hands made of 5 cards are drawn from a deck of 52 cards. Find the probability that :*
   (a) *2 kings are in the hand*
   (b) *1 six, 1 seven, 1 eight, 1 nine and 1 ten are in the hand*
   (c) *2 red aces and 2 black queens are in the hand*
   (d) *1 black jack and 4 cards having the same value are in the hand*

III. not equally likely outcomes

If an experiment can be repeated a large number of time, n, and we **record** the number of experiments, r, say, in which the event A occurs, then $\frac{r}{n}$ is called the **relative frequency** of A.

If this ratio $\frac{r}{n}$ tends to a limit as n approaches infinity, then this limit is the probability $P(A)$.

<u>Exercise</u>

5. *Build an 8-sided spinner, try and make it fair.*
   State upon a number of experiment that would produce an approximation of the probabilities of each outcome, and run them.

IV. Conditional probabilities

If we are interested in the probability that B occurs in those experiments in which A is known to have occured, the probability is called the **conditional probability of B given A**, and is written as $P(B\mid A)$.

If events A and B do not affect each other, they are **independent**, and $P(B\mid A) = P(B)$.

The multiplication rule of probability is really :

$$P(A \text{ and } B) = P(A) \times P(B\mid A)$$

<u>Vocabulary</u> conditional probability of … given ... – independent – to record – relative frequency